24/01/2025
Evening

Memory Based
Answers & Solutions

Time : 3 hrs. *for* **M.M. : 300**

JEE (Main)-2025 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is **Mathematics**. Each part has only two sections: **Section-A** and **Section-B**.

(4) **Section - A :** Attempt all questions.

(5) **Section - B :** Attempt all questions.

(6) **Section - A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

(7) **Section - B (21 – 25) contains** 5 **Numerical value** based questions. The answer to each question should be rounded off to the **nearest integer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A solid sphere and hollow sphere rolls down purely equal distances on same inclined plane (Starting from rest) in time t_1 and t_2 then.

(1) $t_1 > t_2$ (2) $t_1 < t_2$

(3) $t_1 = 2t_2$ (4) $t_1 = t_2$

Answer (2)

Sol. $a_{SS} = \dfrac{g\sin\theta}{1+\frac{2}{5}}$ $a_{HS} = \dfrac{g\sin\theta}{1+\frac{2}{3}}$

$a_{SS} > a_{HS}$ $\Rightarrow$ $t_{SS} < t_{HS}$

2. A solid sphere rolls without slipping on a horizontal plane. What is ratio of translational kinetic energy to the rotational kinetic energy of the sphere.

(1) $\dfrac{4}{3}$

(2) $\dfrac{3}{4}$

(3) $\dfrac{2}{5}$

(4) $\dfrac{5}{2}$

Answer (4)

Sol. $\dfrac{KE_T}{KE_R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}\cdot\frac{2}{5}mR^2\omega^2}$

$= \dfrac{5}{2}$

3. If E, p, m and c denote the energy, linear momentum, mass and speed of light, then the equation representing the correct relation could be

(1) $E^2 = p^2c^2 + m^2c^4$ (2) $E^2 = pc^2 + m^2c^4$

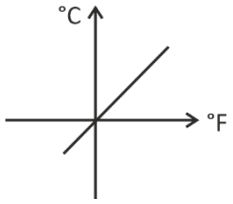
(3) $E = p^2c^2 + m^2c^2$ (4) $E^2 = pc^2 + m^2c^2$

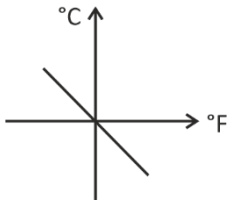
Answer (1)

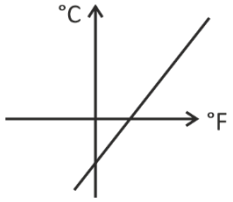
Sol. $[E] = [pc] = [mc^2] = ML^2T^{-2}$

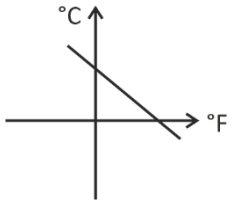
$\Rightarrow$ $[E^2] = [p^2c^2] = [m^2c^4] = [p^2c^2 + m^2c^4]$

4. Which of the following graph correctly represents the relation between Celsius(°c) and Fahrenheit(°F)

(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. $F = \dfrac{9}{5}C + 32$

$C = \dfrac{5}{9}F - \dfrac{5}{9}\times 32$

5. Temperature of a body reduces from 40° to 24°C in 4 minutes in surrounding of 16°C. What is the temperature of body after further 4 minutes?

(1) 20°C (2) 22°C

(3) $\frac{56}{3}$°C (4) 17°C

Answer (3)

Sol. Using Newton's law of cooling

$$\left|<\frac{dT}{dt}>\right| = b(<T> - T_0)$$

$$\frac{16}{4} = b(32-16)$$

$$\frac{24-T}{4} = b\left(\frac{T+24}{2} - 16\right)$$

$$\frac{4\times 4}{24-T} = \frac{16\times 2}{T+24-32}$$

$$48 - 2T = T - 8$$

$$\frac{56}{3} = T$$

6. The position of a particle varies with time as $\vec{r} = (5t^2\hat{i} - 5t\hat{j})$ m. The magnitude and direction of velocity at $t = \frac{1}{2}$ s is

(1) $5\sqrt{2}$ m/s, –45° with +*X* axis

(2) 5 m/s, –45° with +*X* axis

(3) $5\sqrt{2}$ m/s, –45° with +*Y* axis

(4) 5 m/s, +45° with +*Y* axis

Answer (1)

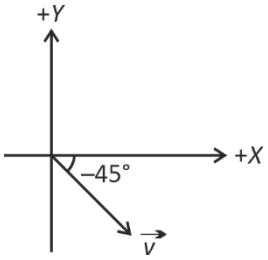
Sol. $\vec{r} = 5t^2\hat{i} - 5t\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 10t\hat{i} - 5\hat{j}$$

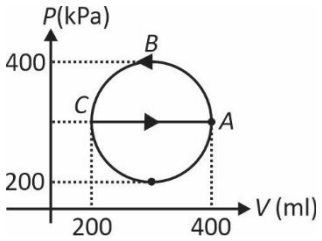
$$\vec{v}\left(t = \frac{1}{2}\text{ s}\right) = 10\left(\frac{1}{2}\right)\hat{i} - 5\hat{j}$$

$$= 5\hat{i} - 5\hat{j}$$

Magnitude = $\sqrt{(5)^2 + (-5)^2} = 5\sqrt{2}$ m/s



7. In given thermodynamic process (Circular in nature), find magnitude of work done by the gas in cycle *ABCA*



(1) 2π (2) 10π

(3) 5π (4) Zero

Answer (3)

Sol. $\omega = \frac{\pi r^2}{2} = \frac{\pi\times(100\times10^3)\times100\times10^{-6}}{2}$

$= 5\pi$

8. Arrange the following in order of decreasing wavelength.

a : Microwave

b : Ultraviolet

c : Infrared

d : X-rays

(1) a > b > c > d

(2) d > c > b > a

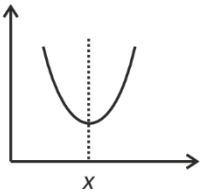
(3) a > c > b > d

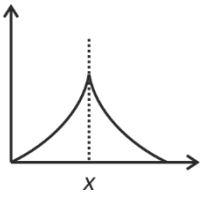
(4) c > a > b > d

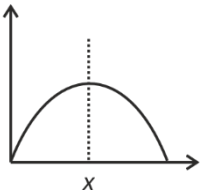
Answer (3)

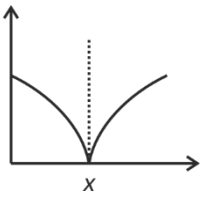
Sol. Theoretical

9. A particle oscillates along x-axis according to law $x = x_0\sin^2(t/2)$ where $x_0 = 1$. Variation of kinetic energy (k) with position (x) is given by graph

(1) 

(2) 

(3) 

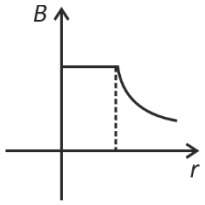
(4) 

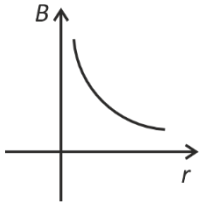
Answer (3)

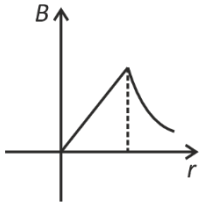
Sol. $x = \sin^2\frac{t}{2} = \frac{1}{2} - \frac{\cos t}{2} \Rightarrow \left(x - \frac{1}{2}\right) = -\frac{\cos t}{2}$

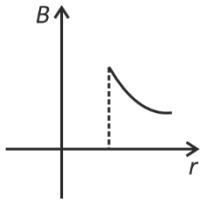
x = 0, 1 are the extremes and $x = \frac{1}{2}$ is the mean position.

10. There is a line solid cylinder carrying current along the axis with uniform current density. Variation of magnetic field (B) with radial distance from axis of cylinder (r) is best denoted by

(1) 

(2) 

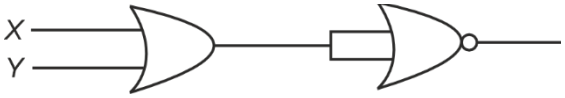
(3) 

(4) 

Answer (3)

Sol. $B = \frac{\mu_0 jr}{2}$ inside and $B = \frac{\mu_0 I}{2\pi r}$ for outside

11. For which of the following input, output of the circuit is zero



(A) $x = 0, y = 0$
(B) $x = 0, y = 1$
(C) $x = 1, y = 0$
(D) $x = 1, y = 1$

(1) A only
(2) A, B, C only
(3) B, C D only
(4) A and C

Answer (3)

Sol. Combination of OR gate and NOT gate.

$\Rightarrow$ Equivalent to NOR gate

12. There is a conical pendulum of mass m and length l making 60° with vertical. Then tension in thread is

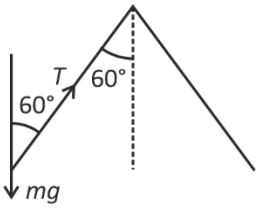
(1) mg (2) $\frac{mg}{2}$

(3) $2mg$ (4) $3mg$

Answer (3)

Sol. $T\cos 60° = mg$

$T = 2mg$



13. There are two identical conducting spheres placed at certain distance l. On of them is carrying charge of 4×10^{-8} C and the other is neutral. Now both are connected using a conducting wire and force between them is found to be 9×10^{-3} N, then distance l is

(1) 4 cm

(2) 4 m

(3) 2 cm

(4) 1 cm

Answer (3)

Sol. $\frac{\theta^0}{4\pi\varepsilon_0 l^2} = F$

$\frac{\left(2\times10^{-8}\right)^2 \times 9\times10^9}{l^2} = 9\times10^{-3}$

$l = 2 \times 10^{-2}$ m

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If the acceleration due to gravity on the surface of earth is 'g', then acceleration due to gravity on a planet whose diameter is $\frac{1}{3}$ of that of earth and same mass as that of earth is $g' = ng$ where n is _____ .

Answer (9)

Sol. Diameter $= 2R = d$

$g = \frac{GM}{R^2} = \frac{4GM}{d^2}$

$g' = \frac{4GM}{(d/3)^2} = 9g$

22. The excess pressure required to decrease the volume of water sample by 0.2% is $P \times 10^5$ Pa. If the bulk modulus of water is 1.25×10^9 Pa, then the value of P is _____ .

Answer (25)

Sol. Given $\frac{\Delta v}{v} \times 100 = -0.2$

$B = -\frac{\Delta P}{(\Delta P / v)}$ or $\Delta p = -B\left(\frac{\Delta v}{v}\right)$

$\Rightarrow \quad \Delta p = -(1.25\times10^9)\left(-\frac{0.2}{100}\right)$ Pa

$= 25 \times 10^5$ Pa

23.

24.

25.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Consider the following reactions

$$S(s)+\frac{3}{2}O_2(g)\rightarrow SO_3(g)+2x\ kJ$$

$$SO_2(g)+\frac{1}{2}O_2(g)\rightarrow SO_3(g)+y\ kJ$$

Calculate ΔH_r for following reaction based on above information

$$S(s)+O_2\rightarrow SO_2(g)$$

(1) $-(x+y)$ (2) $-(2x+y)$

(3) $\frac{x}{y}$ (4) $y-2x$

Answer (4)

Sol. $\Delta H_1 = -2x$ kJ

$\Delta H_2 = -y$ kJ

$\Delta Hr = \Delta H_1 - \Delta H_2$

$\Delta H_r = -2x + y$

2. The conditions and consequences that favour the $t_{2g}^3e_g^1$ configuration in a metal complex are

(1) Strong field ligand; High spin complex

(2) Weak field ligand; High spin complex

(3) Strong field ligand; Low spin complex

(4) Weak field ligand; Low spin complex

Answer (2)

Sol. The electronic configuration of $t_{2g}^3e_g^1$ for central metal ion in a complex implies $\Delta_0 < P$ i.e., weak field ligand and High spin complex.

3. When ethane-1, 2-diammine is progressively added to aqueous solution of Nickel (II) chloride, the sequence of colour change observed will be:

(1) Pale Blue $\rightarrow$ Blue $\rightarrow$ Green $\rightarrow$ Violet

(2) Violet $\rightarrow$ Blue $\rightarrow$ Pale Blue $\rightarrow$ Green

(3) Pale Blue $\rightarrow$ Blue $\rightarrow$ Violet $\rightarrow$ Green

(4) Green $\rightarrow$ Pale Blue $\rightarrow$ Blue $\rightarrow$ Violet

Answer (4)

Sol. If the bidentate ligand, ethane-1, 2-diammine(en) is progressively added in the molar ratio en: Ni 1 : 1, 2 : 1, 3 : 1, the following series of reactions and their associated colour changes occur:-

$$\underset{(Green)}{\left[Ni(H_2O)_6\right]^{2+}}(aq.)+(en)(aq.)\longrightarrow\underset{(Pale\ Blue)}{\left[Ni(H_2O)_4(en)\right]^{2+}}(aq.)+2H_2O$$

$$\underset{(Pale\ Blue)}{\left[Ni(H_2O)_4(en)\right]^{2+}}(aq.)+(en)(aq.)\longrightarrow\underset{(Blue/Purple)}{\left[Ni(H_2O)_2(en)_2\right]^{2+}}(aq.)+2H_2O$$

$$\underset{Blue/Purple}{\left[Ni(H_2O)_2(en)_2\right]^{2+}}(aq.)+(en)(aq.)\longrightarrow\underset{Violet}{\left[Ni(en)_3\right]^{2+}}(aq.)+2H_2O$$

4. S-I :- The first ionisation energy of Pb is greater than that of Sn.

S-II:- The first ionisation energy of Ge is greater than that of Si.

(1) S-I and S-II both are correct

(2) S-I is correct and S-II incorrect

(3) S-I is incorrect and S-II correct

(4) S-I and S-II both are incorrect

Answer (2)

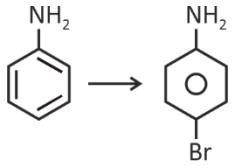
Sol. IE_1 of Pb = 715 kJ/mol

IE_1 of Sn = 708 kJ/mol

IE_1 of Si = 786 kJ/mol

IE_1 of Ge = 761 kJ/mol

SI is correct and S-II is incorrect

5. 

Above conversion can be done by using which reagents among the following.

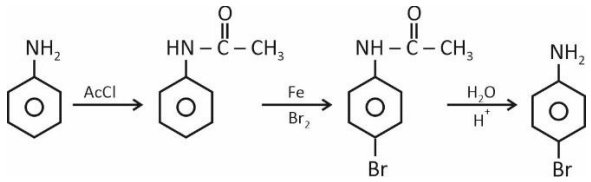
(1) Fe/Br_2, $H_2O(\Delta)$, H_2SO_4

(2) AcOH, H_2SO_4, Br_2, NaOH

(3) AcCl, Fe/Br_2, H_2O/H^+

(4) AcOH, Br_2/Fe, NaOH

Answer (3)

Sol. 

6. Match Column-I with the Column-II and select the correct option.

	Column-I (Ionic species)		**Column-II (Spin only magnetic moment (BM))**
A.	Sc^{3+}	(P)	2.84
B.	Ti^{2+}	(Q)	0
C.	V^{2+}	(R)	5.92
D.	Mn^{2+}	(S)	3.87

(1) A → (P), B → (Q), C → (R), D → (S)

(2) A → (R), B → (S), C → (P), D → (Q)

(3) A → (Q), B → (P), C → (S), D → (R)

(4) A → (Q), B → (P), C → (R), D → (S)

Answer (3)

Sol. $Sc^{3+} \rightarrow$ 0 BM

$Ti^{2+} \rightarrow$ 2.84 BM

$V^{2+} \rightarrow$ 3.87 BM

$Mn^{2+} \rightarrow$ 5.92 BM

7. If a compound contains 54.2% carbon, 9.2% hydrogen and the rest is oxygen. What is molecular formula of the compound, if molecular mass is 132 g/mol?

(1) $C_6H_{12}O_3$

(2) $C_4H_{12}O_3$

(3) $C_4H_{12}O_6$

(4) $C_6H_{13}O_6$

Answer (1)

Sol. Let mass of compound be 100 g

	Mass (g)	**Mole**	**Molar ratio**
C	54.2	$\frac{54.2}{12} = 4.52$	2
H	9.2	$\frac{9.2}{1} = 9.2$	4
O	36.6	$\frac{36.6}{16} = 2.3$	1

Empirical formula = C_2H_4O

$\Rightarrow$ MF = n(EF)

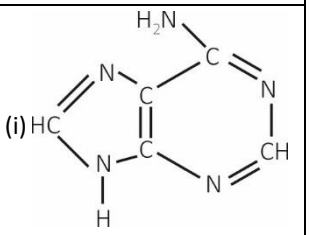
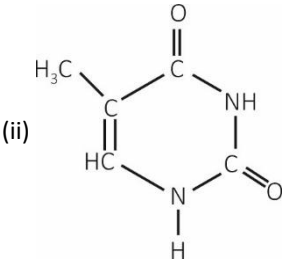
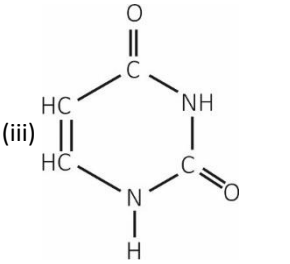
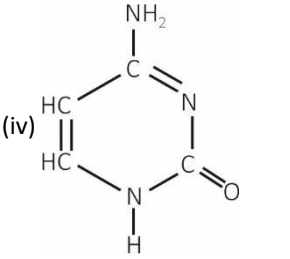
$$n = \frac{\text{MF mass}}{\text{EF mass}}$$

$$\frac{132}{44} = 3$$

MF = $3(C_2H_4O)$

= $C_6H_{12}O_3$

8. Match the following nitrogenous bases present in List-I with their structures present in List-II.

List-I	List-II
A. Thymine	(i) 
B. Adenine	(ii) 
C. Cytosine	(iii) 
D. Uracil	(iv) 

(1) A-(i), B-(ii), C-(iii), D-(iv)

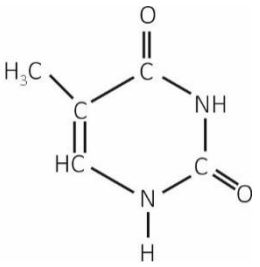
(2) A-(ii), B-(i), C-(iv), D-(iii)

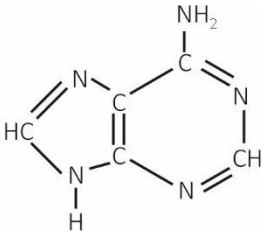
(3) A-(ii), B-(i), C-(iii), D-(iv)

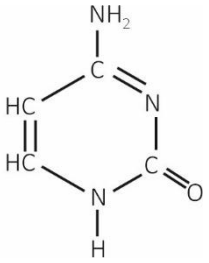
(4) A-(iii), B-(iv), C-(i), D-(ii)

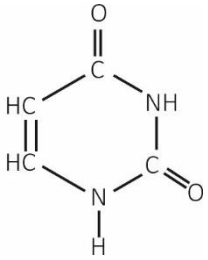
Answer (2)

Sol. Correct structure of

Thymine ⇒ 

Adenine ⇒ 

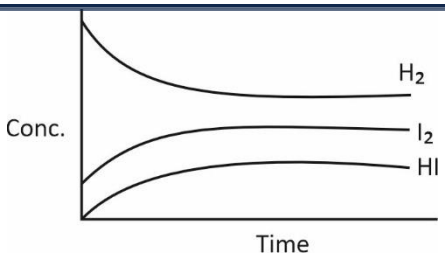
Cytosine ⇒ 

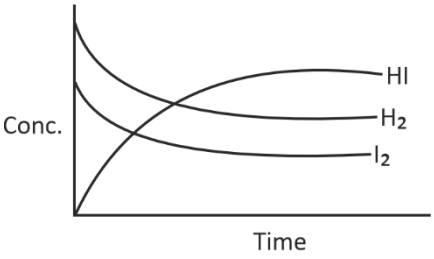
Uracil ⇒ 

9. Consider the following gaseous reaction

$$H_2(g) + I_2(g) \longrightarrow 2HI$$

The above reaction is started with 'a' moles of H_2 and 'b' moles of I_2 in a closed container at a certain temperature T(K) till the equilibrium is established. Which one of the following plots correctly describes the progress of reaction?

(1)


(2)


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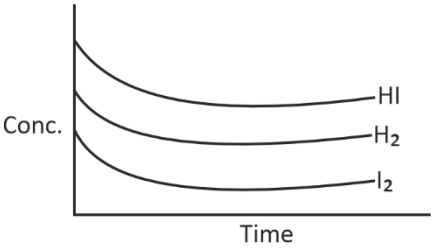

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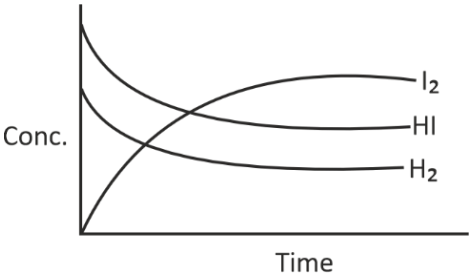
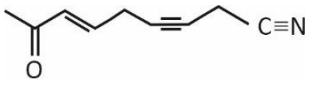


Answer (2)

Sol. The reaction is started with certain concentrations of H_2 and I_2 to form HI. The concentrations of H_2 and I_1 decrease with time while the concentration of HI increases with time till their concentrations become constant at equilibrium.

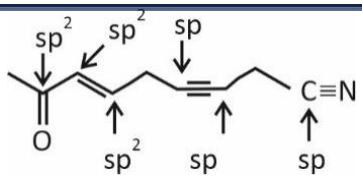
10. In the given compound no. of sp and sp^2 hybridised carbon are



(1) 4 and 5 (2) 4 and 6

(3) 3 and 6 (4) 3 and 3

Answer (4)

Sol.


11. The successive ionisation energy (I.E.) of an element 'X' is given

	$I.E_1$	$I.E_2$	$I.E_3$	$I.E_4$	$I.E_5$
X →	500	600	2000	2200	2600

Data given in KJ/mol.

Find out the group number of element X.

(1) Group → 3 (2) Group → 14

(3) Group → 2 (4) Group → 13

Answer (3)

Sol. Since the ratio of $\frac{IE_3}{IE_2}$ is maximum, so the element X belongs to group 2.

12. Consider the following statements :

Statement-I : Oxygen-oxygen bond in O_3 is greater than O_2.

Statement-II : O – O bond order in O_3 is 1.5 and O – O bond order in O_2 is 2.

(1) Both Statement-I and Statement-II are correct

(2) Both Statement-I and Statement-II are incorrect

(3) Statement-I is correct, Statement-II is incorrect

(4) Statement-I is incorrect, Statement-II is correct

Answer (1)

Sol. $O=\overset{+}{O}-O^{-} \leftrightarrow {}^{-}O-\overset{+}{O}=O$

Bond order = $\frac{3}{2}=1.5$

O = O

Bond order = 2

13.

14.

15.

16.

17.

18.

19.

20.

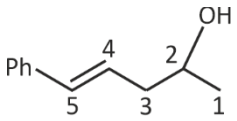
SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. How many stereoisomers are possible for 5-Phenylpent-4-en-2-ol.

Answer (4)

Sol.



There are two centre which can show stereoisomers, one chiral centre and one geometrical centre.

For unsymmetrical compound

Total isomers = 2^n

n = 2

$= 2^2$

= 4

22. A hydrocarbon X which has molar mass 80 g contains 90% carbon by mass. Find degree of unsaturation in X

Answer (3)

Sol. Mass of C-atom $= \frac{90}{100} \times 80$ g

= 72 g

Moles of C-atom $\frac{72}{12}$ = 6 mol C

Mass of H-atom = $\frac{10}{100} \times 80$ = 8 g

Moles of H-atom = $\frac{8}{1}$ = 8 mol H

$\therefore$ Molecular formula of X $\rightarrow C_6H_8$

$D.U = C + 1 - \frac{H}{2}$

$= 6 + 1 - \frac{8}{2}$

= 7 – 4 = 3

Degree of unsaturation $\rightarrow$ 3

23. In Carius method of estimation of halogen, 0.25 g of an organic compound gave 0.16 g of AgBr. What is the percentage of bromine in the organic compound (Given molar mass of Ag = 108, Br = 80)

Answer (27)

Sol. Moles of AgBr $= \frac{0.16}{188}$ moles

Mass of Br $= \frac{0.16}{188} \times 80$ g

= 0.068 g

% of Br $= \frac{0.068}{0.25} \times 100$

= 27%

24. Let k_1, k_2 and k_3 be the rate constant of reaction and $k = \sqrt{\frac{k_1 k_3}{k_2}}$. Then find activation energy of overall reaction. (Given : $E_{a_1} = 10$ kJ/mol, $E_{a_2} = 30$ kJ/mol, $E_{a_3} = 60$ kJ/mol)

Answer (20)

Sol. $(E_a)_{overall} = \frac{1}{2}[E_{a_1} + E_{a_3} - E_{a_2}]$

$= \frac{1}{2}[10 + 60 - 30]$

= 20 kJ/mole

25.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $7 = 5 + \frac{1}{7}(5+\alpha) + \frac{1}{7^2}(5+2\alpha) + \ldots \infty$ terms, then α is equal to

(1) 6 (2) $\frac{6}{7}$

(3) $\frac{1}{7}$ (4) 1

Answer (1)

$S = a + (a + d)r + (a + 2d)r^2 + \ldots \infty$

Then $S = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

$$7 = \frac{S}{1-\frac{1}{7}} + \frac{\alpha \cdot \frac{1}{7}}{\left(1-\frac{1}{7}\right)^2}$$

$\Rightarrow \alpha = 6$

2. If A and B are binomial coefficients of 30^{th} and 12^{th} term of binomial expansion $(1 + x)^{2n-1}$. If $2A = 5B$, then the value of n is

(1) 20 (2) 21

(3) 14 (4) 20

Answer (2)

Sol. $T_{r+1} = {}^{2n-1}C_r \, x^r$

Coefficient of $T_{30} = {}^{2n-1}C_{29} = A$

Coefficient of $T_{12} = {}^{2n-1}C_{11} = B$

$\Rightarrow 2\left({}^{2n-1}C_{29}\right) = 5\left({}^{2n-1}C_{11}\right)$

$\Rightarrow$ Solving we get $n = 21$

3. The equation of chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with (3, 1) as mid-point is

(1) $48x + 25y - 169 = 0$ (2) $25x + 5y - 125 = 0$

(3) $65x + 2y - 12 = 0$ (4) $45x + 4y - 135 = 0$

Answer (1)

Sol. Chord with given mid-point

$\Rightarrow T = S_1$

$$\Rightarrow \left(\frac{xx_1}{25} + \frac{yy_1}{16} - 1\right) = \frac{x_1^2}{25} + \frac{y_1^2}{16} - 1$$

$$\Rightarrow \frac{3x}{25} + \frac{y}{16} - 1 = \frac{9}{25} + \frac{1}{16} - 1$$

$\Rightarrow 48x + 25y - 400 = 144 + 25 - 400$

$\Rightarrow 48x + 25y = 169$

4. If system of equations

$x + 2y - 3z = 2$

$2x + \lambda y + 5z = 5$

$4x + 3y + \mu z = 33$ has infinite solutions, then $\lambda + \mu$ is equal to

(1) $\frac{1334}{5}$ (2) $\frac{1269}{5}$

(3) $\frac{261}{5}$ (4) $\frac{1063}{5}$

Answer (1)

Sol. $\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 4 & 3 & \mu \end{vmatrix} = 0$

$\Rightarrow 12\lambda + \lambda\mu - 4\mu + 7 = 0$

$\Delta z = 25\lambda - 95 = 0$

$\Rightarrow \lambda = \frac{95}{25} = \frac{19}{5}$

$\Delta y = 0$

$\Rightarrow \frac{263-\mu}{5}=0$

$\Rightarrow \mu = 263$

$\therefore \lambda+\mu=\frac{1334}{5}$

5. Let S_n denotes the sum of the first n terms of an arithmetic progression. If $S_{40} = 1030$ and $S_{12} = 57$, then the value of $S_{30} - S_{10}$ is

(1) 505 (2) 510

(3) 515 (4) 520

Answer (3)

Sol. $S_{40} = 1030 \Rightarrow \frac{40}{2}[2a+39d]=1030$ ...(i)

$S_{12} = 57 \Rightarrow \frac{12}{2}[2a+11d] = 57$ ...(ii)

From (i) & (ii) $a = \frac{-7}{2}$, $d = \frac{3}{2}$

$\Rightarrow$

$S_{30} - S_{10} = \frac{30}{2}[2a+29d]-\frac{10}{2}[2a+9d] = 20a+390d$

$= 515$

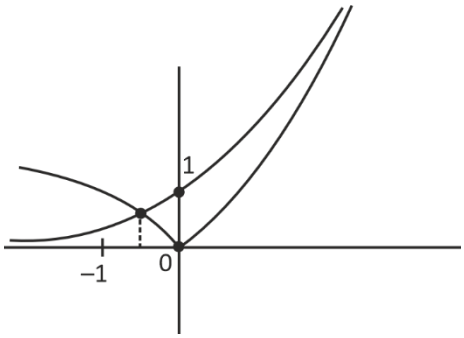
6. The area of region enclosed by the curves $y = e^x, y = |e^x - 1|$ and y – axis is (in sq. units)

(1) 1 (2) 1 – ln2

(3) 1 + ln2 (4) ln2

Answer (2)

Sol.



$e^x = 1 - e^x \Rightarrow 2e^x = 1$

$\Rightarrow e^x = \frac{1}{2}$

$\Rightarrow x = \ln\frac{1}{2}$

$\Rightarrow \int_{\ln\left(\frac{1}{2}\right)}^{0}\left[e^x-(1-e^x)\right]dx$

$=\int_{\ln 2}^{0}(2e^x-1)dx = 2e^x - x\Big|_{-\ln 2}^{0}$

$= 2–(1 + \ln 2)$

$= 1 – \ln 2$

7. Consider an event E such that a matrix of order 2 × 2 is invertible with entries 0 or 1. Then, $P(E)$ is (where $P(X)$ denotes the probability of event X)

(1) $\frac{5}{8}$ (2) $\frac{3}{8}$

(3) $\frac{1}{8}$ (4) $\frac{7}{8}$

Answer (2)

Sol. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$

$\Rightarrow ad - bc = 0$

$\Rightarrow ad = bc$

Case-I $ad = bc = 1$

$a = b = c = d = 1$

Case-II $ad = bc = 0$

$a = 0, d = 0$	$b = 0, c = 0$
$a = 0, d = 1$	$b = 0, c = 1$
$a = 1, d = 0$	$b = 1, c = 0$

$3 \times 3 = 9$ cases

$\therefore \frac{2^4-10}{2^4}=\frac{6}{16}=\frac{3}{8}$

8. The number of real roots of the equation

$x^2+3x+2=\min(|x+2|,|x-3|)$ is

(1) 0

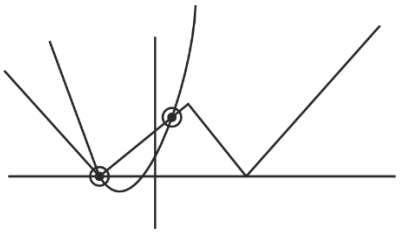
(2) 1

(3) 2

(4) 3

Answer (3)

Sol.



9. A function $f: R \to (-1, 1)$ such that $f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$. The function f is

(1) Both one-one and onto

(2) only one-one

(3) only onto

(4) Both many-one and onto

Answer (1)

Sol. $f(x)=\frac{2^{2x}-1}{2^{2x}+1}=1-\frac{2}{2^{2x}+1}$

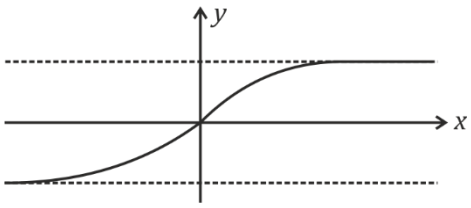
$f'(x)=\frac{2}{(2^{2x}+1)^2}\cdot 2^{2x}\cdot(\ln 2)\cdot 2>0$

$\therefore$ $f(x)$ is increasing $\therefore$ one-one

as $x \to -\infty, f(x) \to -1$

as $x \to \infty, f(x) \to 1$

$\therefore$ range is $(-1, 1)$ $\therefore$ onto



10. Let $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$, $b = \vec{a} \times (\hat{i} - 2\hat{j})$ and $\vec{c} = b \times \hat{k}$, then projection of $\vec{c} - 2\hat{j}$ on $\vec{a}$ is equal to

(1) $2\sqrt{14}$

(2) $3\sqrt{7}$

(3) $2\sqrt{7}$

(4) $\frac{3\sqrt{14}}{14}$

Answer (4)

Sol. $\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ 1 & -2 & 0 \end{vmatrix} = -2\hat{i} - \hat{j} - 8\hat{k}$

$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -1 & -8 \\ 0 & 0 & 1 \end{vmatrix} = -\hat{i} + 2\hat{j}$

$\Rightarrow \vec{c} - 2\hat{j} = -\hat{i}$

$\Rightarrow$ Projection of $\vec{c} - 2\hat{j}$ on $\vec{a} = |(\vec{c} - 2\hat{j}) \cdot \hat{a}|$

$= \frac{3}{\sqrt{14}} = \frac{3\sqrt{14}}{14}$

11. If $\alpha > \beta > \gamma > 0$ then find

$\cot^{-1}\left(\frac{1+\alpha\beta}{\alpha-\beta}\right)+\cot^{-1}\left(\frac{1+\beta\gamma}{\beta-\gamma}\right)+\cot^{-1}\left(\frac{1+\gamma\alpha}{\gamma-\alpha}\right)$

(1) π (2) 0

(3) $\frac{\pi}{2}-(\alpha+\beta+\gamma)$ (4) 3π

Answer (1)

Sol. $\cot^{-1}\left(\frac{1+\alpha\beta}{\alpha-\beta}\right)+\cot^{-1}\left(\frac{1+\beta\gamma}{\beta-\gamma}\right)+\cot^{-1}\left(\frac{1+\gamma\alpha}{\gamma-\alpha}\right)$

$= \tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right)+\pi+\tan^{-1}\left(\frac{\gamma-\alpha}{1+\gamma\alpha}\right)$

$\because \gamma - \alpha < 0$

$= \tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right)+\tan^{-1}\left(\frac{\gamma-\alpha}{1+4\alpha}\right)+\pi$

$= \tan^{-1}\alpha - \tan^{-1}\beta + \tan^{-1}\beta - \tan^{-1}\gamma + \tan^{-1}\gamma - \tan^{-1}\alpha + \pi$

$= 0 + \pi$

$= \pi$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If $\lim\limits_{x\to 0}\begin{vmatrix} a+\frac{\sin x}{x} & 1 & 1 \\ a & 1+\frac{\sin x}{x} & 1 \\ a & 1 & a+\frac{\sin x}{x} \end{vmatrix}$

$= \mu a^2 + \lambda a + \alpha$, then $\mu + \lambda + \alpha$ is

Answer (4)

Sol. $\begin{vmatrix} a+1 & 1 & 1 \\ a & 2 & 1 \\ a & 1 & a+1 \end{vmatrix} = a^2 + 2a + 1$

$\therefore \quad \mu = 1$

$\lambda = 2$

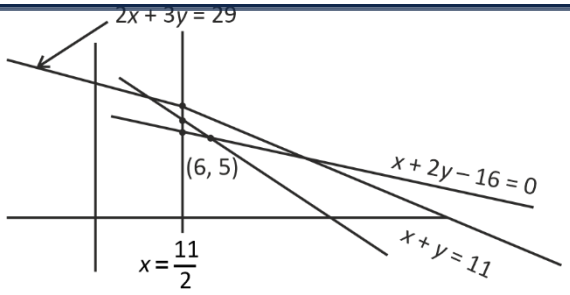
$\alpha = 1$

$\therefore \quad \mu + \lambda + \alpha = 4$

22. The point $P\left(\frac{11}{2}, \alpha\right)$ lies on or inside the triangle formed by the lines $x + y = 11$, $x + 2y = 16$ and $2x + 3y = 29$, then minimum value of 10α is equal to

Answer (55)

Sol.



Clearly, $x = \frac{11}{2}$ intersect

$x + y - 11 = 0$ at $\left(\frac{11}{2}, \frac{11}{2}\right)$ and

$2x + 3y - 29 = 0$ at $\left(\frac{11}{2}, 6\right)$.

$\Rightarrow \quad \alpha \in \left[\frac{11}{2}, 6\right]$

$\Rightarrow$ minimum value of $10\alpha = 55$

23. If $\int \frac{2x^2+5x+9}{x^2+x+1}dx = x\sqrt{x^2+x+1} + \alpha\sqrt{x^2+x+1} + \beta \ln\left(x + \frac{1}{2} + \sqrt{x^2+x+1}\right) + C$, then $\alpha + 2\beta$ equals to

Answer (16)

Sol. $I = \int \frac{2x^2+5x+9}{x^2+x+1}dx.$

Let $\frac{2x^2+5x+9}{\sqrt{x^2+x+1}} = \frac{A\left(x^2+x+1\right) + B(2X+1) + C}{\sqrt{x^2+x+1}}$

Then, $A = 2$, $B = \frac{3}{2}$ and $C = \frac{11}{2}$

$\therefore \; I = \int \frac{2\left(x^2+x+1\right) + \frac{3}{2}(2x+1) + \frac{11}{2}}{\sqrt{x^2+x+1}}dx$

$= 2\int \sqrt{x^2+x+1}\,dx + \frac{3}{2}\cdot 2\sqrt{x^2+x+1} + \frac{11}{2}\int \frac{1}{\sqrt{x^2+x+1}}dx$

$$= 2\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx + 3\sqrt{x^2+x+1} + \frac{11}{2}\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= 2\left\{\frac{x+\frac{1}{2}}{2}\sqrt{x^2+x+1}+\frac{3}{8}\ln\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|\right\} + 3\sqrt{x^2+x+1}+\frac{11}{2}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$$= \left(\frac{2x+1}{2}\right)\sqrt{x^2+x+1}+\frac{3}{4}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right| + 3\sqrt{x^2+x+1}+\frac{22}{7}\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$$= \frac{2x+7}{2}\sqrt{x^2+x+1}+\frac{25}{4}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$$= x\sqrt{x^2+x+1}+\frac{7}{2}\sqrt{x^2+x+1} + \frac{25}{4}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

$\therefore\ \alpha=\frac{7}{2},\ \beta=\frac{25}{4}$

Then $\alpha+2\beta=\frac{7}{2}+\frac{25}{2}-16$

24. In group *A*, there are 7 boys and 3 girls and in group *B*, there are 4 boys and 5 girls. For a picnic trip 4 boys and 4 girls are required such that 5 are selected from group A and 3 are selected from group *B*. Then the total number of ways to select the team for picnic trip is

Answer (5880)

Sol. Group A (7B + 3G), Group B(4B + 5G)

Number of required ways = $({}^7C_4\cdot{}^3C_1)\cdot({}^4C_0\cdot{}^5C_3) + ({}^7C_3\cdot{}^3C_2)\cdot({}^4C_1\cdot{}^5C_2) + ({}^7C_2\cdot{}^3C_3)\cdot({}^4C_2\cdot{}^5C_1)$

= 5880

25.