Ex - 6.5

Q1. Sides of some triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Sol. (i) $(7)^2 + (24)^2 = 49 + 576 = 625 = (25)^2$

Therefore, given sides 7 cm, 24 cm, 25 cm make a right triangle.

(ii) $(6)^2 + (3)^2 = 36 + 9 = 45$

$(8)^2 = 64$

$(6)^2 + (3)^2 \neq (8)^2$

Therefore, given sides 3cm, 8 cm, 6 cm does not make a right triangle.

(iii) $(50)^2 + (80)^2 = 2500 + 6400 = 8900$

$(100)^2 = 10000$

$(50)^2 + (80)^2 \neq 100^2$

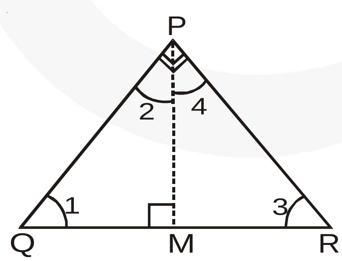
Therefore, given sides 50cm, 80 cm, 100 cm does not make a right triangle.

(iv) $(12)^2 + (5)^2 = 144 + 25 = 169 = (13)^2$

Therefore, given sides 13cm, 12 cm, 5 cm make a right triangle.

Q2. PQR is a triangle right angled at P and M is a point on QR such that PM $\perp$ QR. Show that $PM^2 = QM \times MR$.

Sol. $\angle 1 + \angle 2 = \angle 2 + \angle 4$ (Each = 90°)



$\Rightarrow \angle 1 = \angle 4$

Similarly, $\angle 2 = \angle 3$. Now, this gives

$\Delta QPM \sim \Delta PRM$ (AA similarity)

$\Rightarrow \dfrac{ar(\Delta QPM)}{ar(\Delta PRM)} = \dfrac{PM^2}{RM^2}$ (By theorem 6.7)

$$\Rightarrow \frac{\frac{1}{2}(QM)\times(PM)}{\frac{1}{2}(RM)\times(PM)} = \frac{PM^2}{RM^2} \left(\begin{array}{l}\text{Area of a triangle}\\ = \frac{1}{2}\times\text{Base}\times\text{Height}\end{array}\right)$$

$$\Rightarrow \frac{QM}{RM} = \frac{PM^2}{RM^2}$$

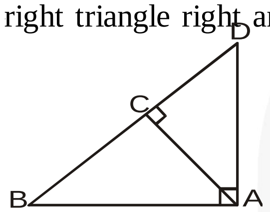
$\Rightarrow$ $PM^2 = QM \times RM$ or $PM^2 = QM \times MR$

Q3. In figure, ABD is a right triangle right angled at A and AC $\perp$ BD. Show that

(i) $AB^2 = BC.BD$

(ii) $AC^2 = BC.DC$

(iii) $AD^2 = BD.CD$



Sol. In the given figure, we have

$\Delta ABC \sim \Delta DAC \sim \Delta DBA$

(i) $\Delta ABC \sim \Delta DBA$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBA)} = \frac{AB^2}{DB^2} \Rightarrow \frac{\frac{1}{2}(BC)\times(AC)}{\frac{1}{2}(BD)\times(AC)} = \frac{AB^2}{DB^2} \Rightarrow \quad AB^2 = BC.BD$$

(ii) $\Delta ABC \sim \Delta DAC$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DAC)} = \frac{AC^2}{DC^2} \Rightarrow \frac{\frac{1}{2}(BC)\times(AC)}{\frac{1}{2}(DC)\times(AC)} = \frac{AC^2}{DC^2} \Rightarrow \quad AC^2 = BC.DC$$

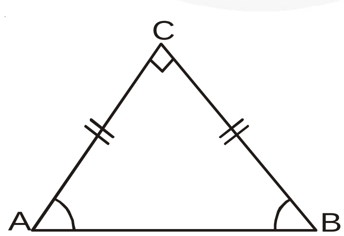
(iii) $\Delta DAC \sim \Delta DBA$

$$\Rightarrow \frac{ar(\Delta DAC)}{ar(\Delta DBA)} = \frac{DA^2}{DB^2} \Rightarrow \frac{\frac{1}{2}(CD)\times(AC)}{\frac{1}{2}(BD)\times(AC)} = \frac{AD^2}{BD^2} \Rightarrow \quad AD^2 = BD.CD$$

Q4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Sol. In ΔABC, $\angle ACB = 90°$. We are given that

ΔABC is an isosceles triangle.



$\Rightarrow$ $\angle A = \angle B = 45°$

$\Rightarrow$ $AC = BC$ ...(1)

By pythagoras theorem, we have

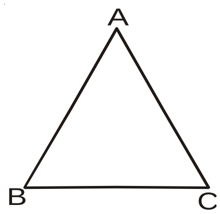
$AB^2 = AC^2 + BC^2$

$= AC^2 + AC^2$ {$\because$ BC = AC by (1)]

$= 2\ AC^2$

Q5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2\ AC^2$, prove that ABC is a right triangle.

Sol.



As, $AB^2 = 2AC^2$

$AB^2 = AC^2 + AC^2$

$= AC^2 + BC^2$ [$\because$ AC = BC]

As it satisfy the pythagoran triplet

So, ΔABC is right triangle, right angled at $\angle C$.

Q6. ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Sol. Altitude of equilateral triangle

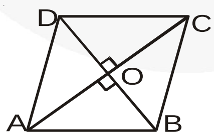
$= \frac{\sqrt{3}}{2} \times \text{Side} = \frac{\sqrt{3}}{2} \times 2a = \sqrt{3}\ a$

Q7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Sol. ABCD is a rhombus in which AB = BC = CD = DA = a (say). Its diagonals AC and BD are right bisectors of each other at O.

In ΔOAB, $\angle AOB = 90°$,

$OA = \frac{1}{2}AC$ and $OB = \frac{1}{2}BD$



By pythagoras theorem, we have

$OA^2 + OB^2 = AB^2$

$\Rightarrow \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 = AB^2$

$\Rightarrow AC^2 + BD^2 = 4\ AB^2$

or $4\ AB^2 = AC^2 + BD^2$

$\Rightarrow$ $AB^2 + BC^2 + CD^2 + DA^2$

$=$ $AC^2 + BD^2$.

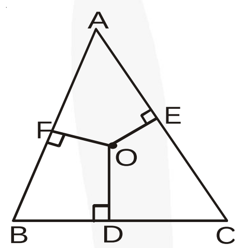
Hence proved.

Q8. In figure, O is a point in the interior of a triangle ABC, OD $\perp$ BC, OE $\perp$ AC and OF $\perp$ AB. Show that

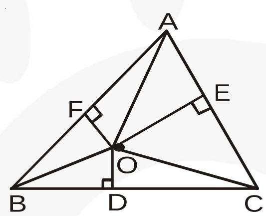
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

$= AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.



Sol. (i) In right angled ΔOFA,



$OA^2 = OF^2 + AF^2$ (Pythagoras theorem)

$\Rightarrow$ $OA^2 - OF^2 = AF^2$ ...(1)

Similarly, $OB^2 - OD^2 = BD^2$ ...(2)

and $OC^2 - OE^2 = CE^2$ ...(3)

Adding (1), (2) and (3), we get

$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

$= AF^2 + BD^2 + CE^2$.

(ii) We have proved that

$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

$= AF^2 + BD^2 + CE^2$. ...(4)

Similarly, we can prove that

$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

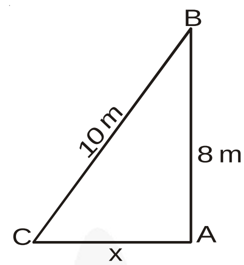
$= BF^2 + CD^2 + AE^2$. ...(5)

From (4) and (5), we have

$AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.

Q9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Sol. Let AC = x metres be the distance of the foot of the ladder from the base of the wall.



AB = 8 m (Height of window)

BC = 10 m (length of ladder)

Now, $x^2 + (8)^2 = (10)^2$

$\Rightarrow x^2 = 100 - 64 = 36 \Rightarrow x = 6$, i.e., AC = 6 m

Q10. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Sol. Let AB be the vertical pole of 18 m and AC be the wire of 24 m.

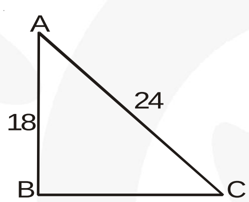
The ΔABC, by pythagoras theorem

$AC^2 = AB^2 + BC^2$

$24^2 = 18^2 + BC^2$

$BC^2 = 252$

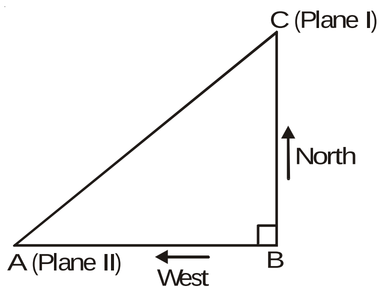
$BC = 6\sqrt{7}$ m



Q11. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Sol. The first plane travels distance BC in the direction of north in $1\frac{1}{2}$ hours at a speed of 1000 km/hr.

$\therefore$ $BC = 1000 \times \frac{3}{2}$ km = 1500 km.



The second plane travels distance BA in the direction of west in $1\frac{1}{2}$ hours at a speed of 1200 km/hr.

$\therefore$ BA = 1200 × $\frac{3}{2}$ km = 1800 km.

From right angled ΔABC,

$AC^2 = AB^2 + BC^2$

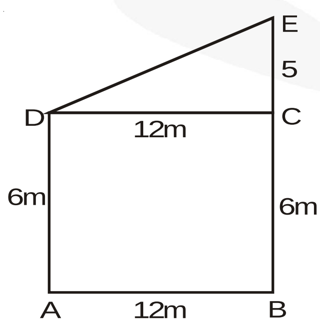
$= (1800)^2 + (1500)^2$

$= 3240000 + 2250000 = 5490000$

$\Rightarrow$ $AC = \sqrt{5490000}\text{ m}$ $\Rightarrow$ $AC = 300\sqrt{61}\text{ m}$

Q12. Two poles of height 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Sol.



Let AD and BE be two poles of height 6 m and 11 m and AB = 12 m

In ΔDEC, by pythagoras theorem

$DE^2 = CD^2 + CE^2$

$DE^2 = 12^2 + 5^2$ (DC = AB = 12 m)

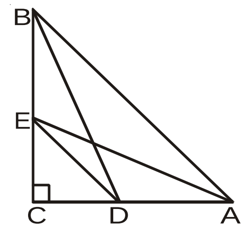
$DE = \sqrt{144+25} = \sqrt{169} = 13$ m

Thus, distance between their tops is 13 m.

Q13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Sol. In right angled ΔACE,

$AE^2 = CA^2 + CE^2$ ...(1)



and in right angled ΔBCD,

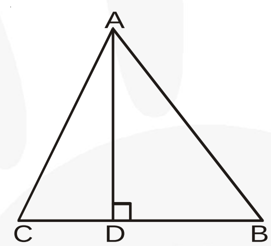
$BD^2 = BC^2 + CD^2$ ...(2)

Adding (1) and (2), we get

$$AE^2 + BD^2 = (CA^2 + CE^2) + (BC^2 + CD^2)$$
$$= (BC^2 + CA^2) + (CD^2 + CE^2)$$
$$= BA^2 + DE^2$$

$\therefore$ $AE^2 + BD^2 = AB^2 + DE^2$

Q14. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3 CD (see figure). Prove that $2\,AB^2 = 2\,AC^2 + BC^2$.



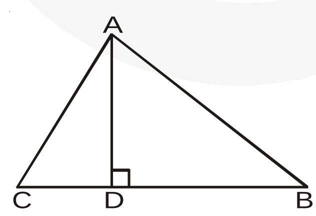
Sol. DB = 3 CD

$\Rightarrow$ $CD = \frac{1}{4}\ BC$ ...(1)

and $DB = \frac{3}{4}\ BC$

In ΔABD, $AB^2 = DB^2 + AD^2$ ...(2)

In ΔACD, $AC^2 = CD^2 + AD^2$ ...(3)



Subtracting (3) from (2), we get

$AB^2 - AC^2 = DB^2 - CD^2$

$$= \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \frac{9}{16}BC^2 - \frac{1}{16}BC^2$$

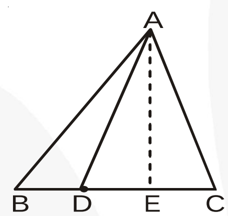
$= \frac{1}{2}BC^2 \Rightarrow \quad 2AB^2 - 2AC^2 = BC^2$

$\Rightarrow \; 2AB^2 = 2AC^2 + BC^2$

Hence proved.

Q15. In an equailateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$BC. Prove that $9AD^2 = 7\ AB^2$.

Sol. AB = BC = CA = a (say)



$BD = \frac{1}{3}BC = \frac{1}{3}a$

$\Rightarrow \; CD = \frac{2}{3}BC = \frac{2}{3}a$

$AE \perp BC$

$\Rightarrow \; BE = EC = \frac{1}{2}a$

$DE = \frac{1}{2}a - \frac{1}{3}a = \frac{1}{6}a$

$AD^2 = AE^2 + DE^2 = AB^2 - BE^2 + DE^2$

$= a^2 - \left(\frac{1}{2}a\right)^2 + \left(\frac{1}{6}a\right)^2$

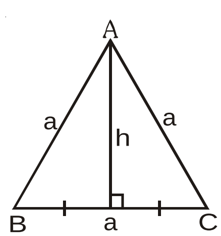
$= a^2 - \frac{1}{4}a^2 + \frac{1}{36}a^2$

$= \frac{(36-9+1)a^2}{36} = \frac{28}{36}a^2 = \frac{7}{9}AB^2$

$\Rightarrow \; 9AD^2 = 7\ AB^2$

Q16. In an equailateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Sol.



Altitude of equilateral $\Delta = \frac{\sqrt{3}}{2}$ side

$h = \frac{\sqrt{3}}{2} a$

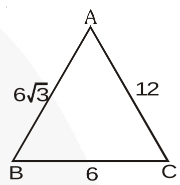
$h^2 = \frac{3}{4} a^2$

$\boxed{4h^2 = 3a^2}$

Q17. Tick the correct answer and justify : In ΔABC, $AB = 6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. The angle B is :

(1) 120° (2) 60° (3) 90° (4) 45°

Sol. $AB^2 = (6\sqrt{3})^2 = 108$



$BC^2 = 6^2 = 36$

$AC^2 = 12^2 = 144$

So, $AB^2 + BC^2 = AC^2$

ΔABC is right Δ, right angled at B

$\angle B = 90°$.