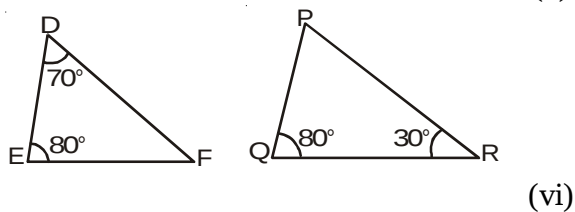
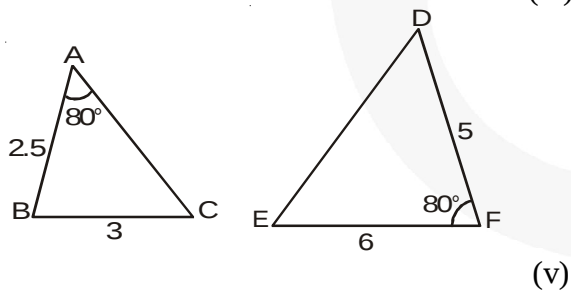
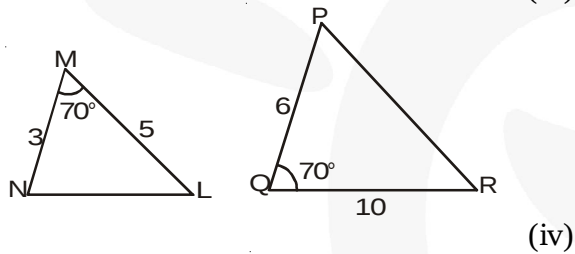
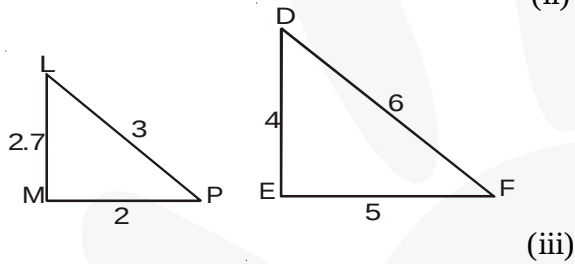
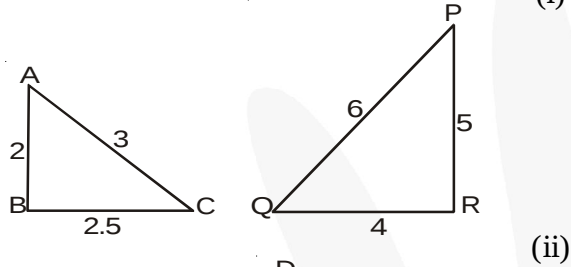
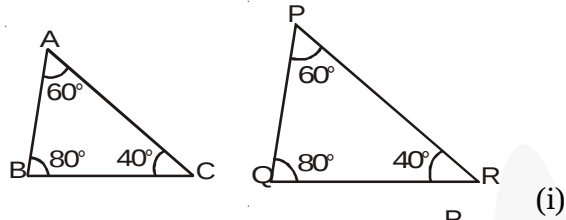


Ex - 6.3

Q1. State which pairs of triangles in figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :



Sol. (i) Yes. $\angle A = \angle P = 60^\circ$, $\angle B = \angle Q = 80^\circ$,

$$\angle C = \angle R = 40^\circ$$

Therefore, $\triangle ABC \sim \triangle PQR$.

By AAA similarity criterion

(ii) Yes.

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}, \frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2}$$

Therefore, $\triangle ABC \sim \triangle QRP$.

By SSS similarity criterion.

(iii) No.

$$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}, \frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}, \frac{LM}{EF} = \frac{2.7}{5} \neq \frac{1}{2}$$

$$\text{i.e., } \frac{MP}{DE} = \frac{LP}{DF} \neq \frac{LM}{EF}$$

Thus, the two triangles are not similar.

(iv) Yes,

$$\frac{MN}{QP} = \frac{ML}{QR} = \frac{1}{2}$$

$$\text{and } \angle NML = \angle PQR = 70^\circ$$

By SAS similarity criterion

$$\triangle NML \sim \triangle PQR$$

(v) No,

$$\frac{AB}{FD} \neq \frac{AC}{FE}$$

Thus, the two triangles are not similar

(vi) In triangle DEF $\angle D + \angle E + \angle F = 180^\circ$

$$70^\circ + 80^\circ + \angle F = 180^\circ$$

$$\angle F = 30^\circ$$

In triangle PQR

$$\angle P + 80^\circ + 30^\circ = 180^\circ$$

$$\angle P = 70^\circ$$

$$\angle E = \angle Q = 80^\circ$$

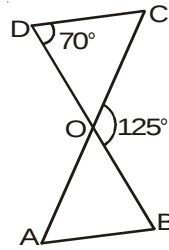
$$\angle D = \angle P = 70^\circ$$

$$\angle F = \angle R = 30^\circ$$

By AAA similarity criterion,

$$\triangle DEF \sim \triangle PQR.$$

Q2. In figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Sol. From figure,

$$\angle DOC + 125^\circ = 180^\circ$$

$$\Rightarrow \angle DOC = 180^\circ - 125^\circ = 55^\circ$$

$$\angle DCO + \angle CDO + \angle DOC = 180^\circ$$

(Sum of three angles of $\triangle ODC$)

$$\Rightarrow \angle DCO + 70^\circ + 55^\circ = 180^\circ$$

$$\Rightarrow \angle DCO + 125^\circ = 180^\circ$$

$$\Rightarrow \angle DCO = 180^\circ - 125^\circ = 55^\circ$$

Now, we are given that $\triangle ODC \sim \triangle OBA$

$$\Rightarrow \angle OCD = \angle OAB$$

$$\Rightarrow \angle OAB = \angle OCD = \angle DCO = 55^\circ$$

i.e., $\angle OAB = 55^\circ$

Hence, we have

$$\angle DOC = 55^\circ, \angle DCO = 55^\circ, \angle OAB = 55^\circ$$

Q3. Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point

O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

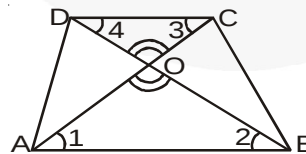
Sol. In figure, $AB \parallel DC$

$$\Rightarrow \angle 1 = \angle 3, \angle 2 = \angle 4$$

(Alternate interior angles)

Also $\angle DOC = \angle BOA$

(Vertically opposite angles)

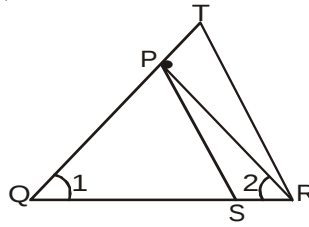


$$\Rightarrow \triangle OCD \sim \triangle OAB \quad \Rightarrow \quad \frac{OC}{OA} = \frac{OD}{OB}$$

(Ratios of the corresponding sides of the similar triangle)

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD} \text{ (Taking reciprocals)}$$

Q4. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Sol. In figure, $\angle 1 = \angle 2$ (Given)

$$\Rightarrow PQ = PR$$

(Sides opposite to equal angles of ΔPQR)

We are given that

$$\frac{QR}{QS} = \frac{QT}{PR}$$

$$\Rightarrow \frac{QR}{QS} = \frac{QT}{PQ} \quad (\because PQ = PR \text{ proved})$$

$$\Rightarrow \frac{QS}{QR} = \frac{PQ}{QT} \quad (\text{Taking reciprocals}) \dots (1)$$

Now, in ΔPQS and ΔTQR , we have

$$\angle PQS = \angle TQR \quad (\text{Each} = \angle 1)$$

$$\text{and } \frac{QS}{QR} = \frac{PQ}{QT} \quad (\text{By (1)})$$

Therefore, by SAS similarity criterion, we have

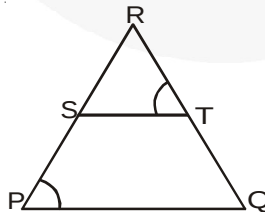
$$\Delta PQS \sim \Delta TQR.$$

Q5. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Sol. In figure, We have ΔRPQ and ΔRTS in which

$$\angle RPQ = \angle RTS \text{ (Given)}$$

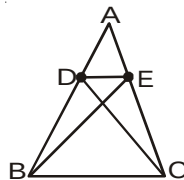
$$\angle PRQ = \angle SRT \text{ (Each} = \angle R)$$



Then by AA similarity criterion, we have

$$\Delta RPQ \sim \Delta RTS$$

Q6. In figure, if $\triangle ABE \cong \triangle ACD$, show that $\triangle ADE \sim \triangle ABC$.



Sol. In figure,

$$\triangle ABE \cong \triangle ACD \quad (\text{Given})$$

$$\Rightarrow AB = AC \text{ and } AE = AD \quad (\text{CPCT})$$

$$\Rightarrow \frac{AB}{AC} = 1 \text{ and } \frac{AD}{AE} = 1$$

$$\Rightarrow \frac{AB}{AC} = \frac{AD}{AE} \quad (\text{Each} = 1)$$

Now, in $\triangle ADE$ and $\triangle ABC$, we have

$$\frac{AD}{AE} = \frac{AB}{AC} \quad (\text{proved})$$

$$\text{i.e., } \frac{AD}{AB} = \frac{AE}{AC}$$

$$\text{and also } \angle DAE = \angle BAC \quad (\text{Each} = \angle A)$$

$$\Rightarrow \triangle ADE \sim \triangle ABC (\text{By SAS similarity criterion})$$

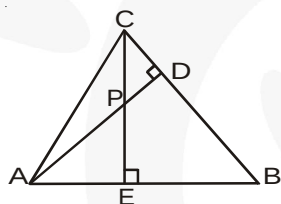
Q7. In figure, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that :

$$(i) \triangle AEP \sim \triangle CDP$$

$$(ii) \triangle ABD \sim \triangle CBE$$

$$(iii) \triangle AEP \sim \triangle ADB$$

$$(iv) \triangle PDC \sim \triangle BEC$$



Sol. (i) In $\triangle AEP$ and $\triangle CDP$,

$$\angle APE = \angle CPD \text{ (vertically opposite angles)}$$

$$\angle AEP = \angle CDP = 90^\circ$$

$\therefore$ By AA similarity

$$\triangle AEP \sim \triangle CDP$$

(ii) In $\triangle ABD$ and $\triangle CBE$,

$$\angle ABD = \angle CBE \text{ (common)}$$

$$\angle ADB = \angle CEB = 90^\circ$$

$\therefore$ By AA similarity

$$\triangle ABD \sim \triangle CBE$$

(iii) In $\triangle AEP$ and $\triangle ADB$,

$$\angle PAE = \angle DAB \text{ (common)}$$

$$\angle AEP = \angle ADB = 90^\circ$$

$\therefore$ By AA similarity

$$\triangle AEP \sim \triangle ADB$$

(iv) In $\triangle PDC$ and $\triangle BEC$,

$$\angle PCD = \angle BCE \text{ (common)}$$

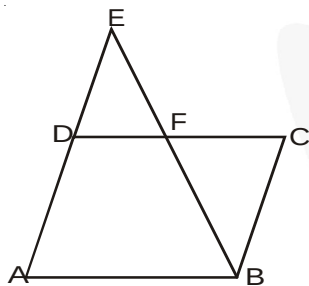
$$\angle PDC = \angle BEC = 90^\circ$$

$\therefore$ By AA similarity

$$\triangle PDC \sim \triangle BEC$$

Q8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$.

Sol.



In $\triangle ABE$ and $\triangle CFB$,

$$\angle EAB = \angle BCF \text{ (opp. angles of parallelogram)}$$

$$\angle AEB = \angle CBF \text{ (Alternate interior angles, As } AE \parallel BC \text{)}$$

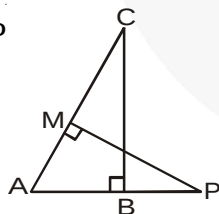
$\therefore$ By AA similarity

$$\triangle ABE \sim \triangle CFB$$

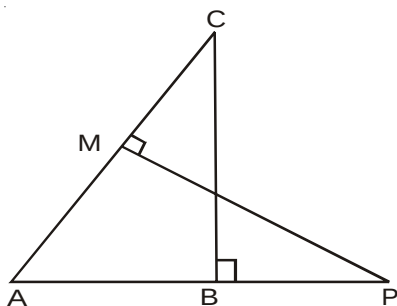
Q9. In figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) $\triangle ABC \sim \triangle AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$



Sol.



(i) In $\triangle ABC$ and $\triangle AMP$

$$\angle CAB = \angle PAM \text{ (common)}$$

$$\angle ABC = \angle AMP = 90^\circ$$

$\therefore$ By AA similarity

$$\triangle ABC \sim \triangle AMP$$

(ii) As $\triangle ABC \sim \triangle AMP$ (Proved above)

$$\therefore \frac{CA}{PA} = \frac{BC}{MP}$$

Q10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle EFG$ respectively. If $\triangle ABC \sim \triangle FEG$, show that :

(i) $\frac{CD}{GH} = \frac{AC}{FG}$ (ii) $\triangle DCB \sim \triangle HGE$

(iii) $\triangle DCA \sim \triangle HGF$

Sol. $\triangle ABC \sim \triangle FEG$

$$\Rightarrow \angle ACB = \angle EGF$$

$$\Rightarrow \frac{1}{2} \angle ACB = \frac{1}{2} \angle EGF$$

$$\Rightarrow \angle DCB = \angle HGE \quad \dots(1)$$

$$\text{Also, } \angle B = \angle E$$

$$\Rightarrow \angle DBC = \angle HEG \quad \dots(2)$$

From (1) and (2), we have

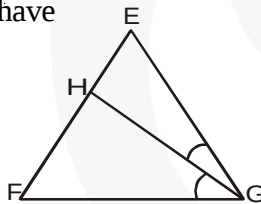
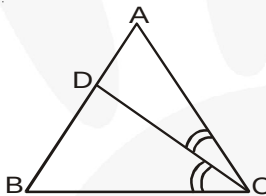
$$\Rightarrow \triangle DCB \sim \triangle HGE$$

Similarly, we have

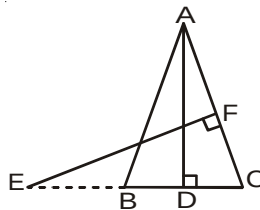
$$\triangle DCA \sim \triangle HGF$$

Now, $\triangle DCA \sim \triangle HGF$

$$\Rightarrow \frac{DC}{HG} = \frac{CA}{GF} \Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$



Q11. In figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.



Sol. In figure,

We are given that $\triangle ABC$ is isosceles.

and $AB = AC$

$$\Rightarrow \angle B = \angle C \dots(1)$$

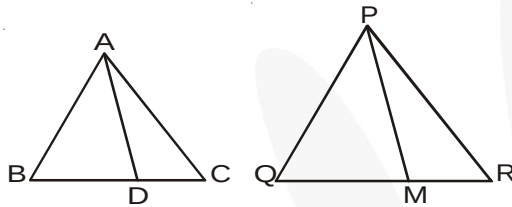
For triangles ABD and ECF ,

$$\angle ABD = \angle ECF \quad \{\text{from (1)}\}$$

and $\angle ADB = \angle EFC \quad \{\text{each} = 90^\circ\}$

$$\Rightarrow \triangle ABD \sim \triangle ECF \text{ (AA similarity)}$$

Q12. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR . Show that $\triangle ABC \sim \triangle PQR$.



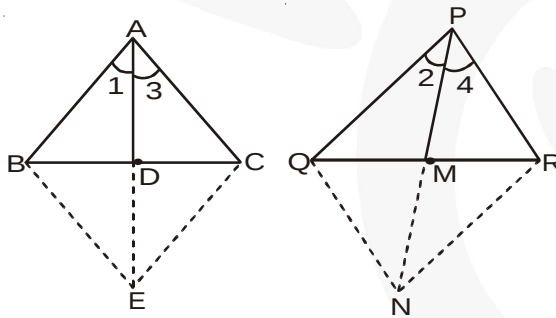
Sol. Given. $\triangle ABC$ and $\triangle PQR$. AD and PM are their medians respectively.

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \dots(1)$$

To prove. $\triangle ABC \sim \triangle PQR$.

Construction : Produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$.

Join BE , CE , QN , RN .



Proof : Quadrilaterals $ABEC$ and $PQNR$ are parallelograms because their diagonals bisect each other at D and M respectively.

$$\Rightarrow BE = AC \text{ and } QN = PR.$$

$$\Rightarrow \frac{BE}{QN} = \frac{AC}{PR} \Rightarrow \frac{BE}{QN} = \frac{AB}{PQ} \quad (\text{By 1})$$

$$\text{i.e., } \frac{AB}{PQ} = \frac{BE}{QN} \dots(2)$$

$$\text{From (1), } \frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$$

$$\text{i.e., } \frac{AB}{PQ} = \frac{AE}{PN} \dots(3)$$

From (2) and (3), we have

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$$\Rightarrow \triangle ABE \sim \triangle PQN \Rightarrow \angle 1 = \angle 2 \quad \dots(4)$$

Similarly, we can prove

$$\Rightarrow \triangle ACE \sim \triangle PRN \Rightarrow \angle 3 = \angle 4 \quad \dots(5)$$

Adding (4) and (5), we have

$$\Rightarrow \angle 1 + \angle 3 = \angle 2 + \angle 4 \Rightarrow \angle A = \angle P$$

$$\Rightarrow \triangle ABC \sim \triangle PQR \text{ (SAS similarity criterion)}$$

Q13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.

Sol. For $\triangle ABC$ and $\triangle DAC$, We have

$$\angle BAC = \angle ADC \quad (\text{Given})$$

$$\text{and } \angle ACB = \angle DCA \quad (\text{Each} = \angle C)$$

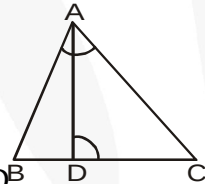
$$\Rightarrow \triangle ABC \sim \triangle DAC \quad (\text{AA similarity})$$

$$\Rightarrow \frac{AC}{DC} = \frac{CB}{CA}$$

$$\Rightarrow \frac{CA}{CD} = \frac{CB}{CA}$$

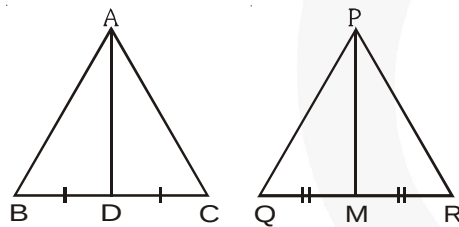
$$\Rightarrow CA \times CA = CB \times CD$$

$$\Rightarrow CA^2 = CB \times CD$$



Q14. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see figure). Show that $\triangle ABC \sim \triangle PQR$.

Sol.



$$\text{As, } \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM} \quad (\text{Given})$$

$$\text{So, } \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$$\left\{ \because \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QM} \right\}$$

$\therefore$ By SSS similarity,

$$\triangle ABD \sim \triangle PQM.$$

$$\text{As, } \triangle ABD \sim \triangle PQM.$$

$$\therefore \angle ABD = \angle PQM$$

Now, In $\triangle ABC$ and $\triangle PQR$

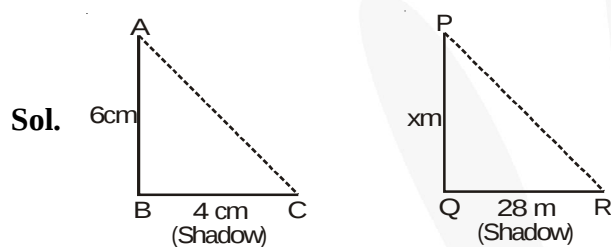
$$\frac{AB}{PQ} = \frac{BC}{QR} \text{ (Given)}$$

$$\angle ABC = \angle PQR \text{ (Proved above)}$$

$\therefore$ By SAS similarity

$$\triangle ABC \sim \triangle PQR.$$

Q15. A vertical stick of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.



$$\triangle ABC \sim \triangle PQR$$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{6}{x} = \frac{4}{28}$$

$$\Rightarrow x = 42 \text{ m}$$

Q16. If AD and PM are medians of triangles ABC and PQR, respectively where $\triangle ABC \sim \triangle PQR$, prove

that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Sol. $\triangle ABC \sim \triangle PQR$ (Given)

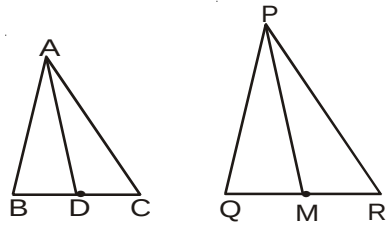
$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR};$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \quad \dots(1)$$

Now, $BD = CD = \frac{1}{2} BC$

and $QM = RM = \frac{1}{2} QR \quad \dots(2)$

($\because$ D is mid-point of BC and M is mid-point of QR)



From (1), $\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$ (By (2))

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$$

Thus, we have $\frac{AB}{PQ} = \frac{BD}{QM}$

and $\angle ABD = \angle PQM$ ($\because \angle B = \angle Q$)

$\Rightarrow \triangle ABD \sim \triangle PQM$ (By SAS similarity criterion)

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$