Ex - 13.3

NOTE: Take $\pi = \frac{22}{7}$, unless stated otherwise.

Q1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Sol. Let the height of the cylinder made be equal to h cm.

Its radius = 6 cm

Volume of the cylinder made = Volume of the given metallic sphere of radius 4.2 cm

$$\Rightarrow \pi \times (6)^2 \times h = \frac{4}{3}\pi \times (4.2)^3 \text{ cm}^3$$

$$\Rightarrow 36 \times h = \frac{4}{3} \times 4.2 \times 4.2 \times 4.2$$

$$\Rightarrow h = \frac{4.2 \times 4.2 \times 4.2}{3 \times 9} \text{ cm} = 1.4 \times 1.4 \times 1.4 \text{ cm} = 2.744 \text{ cm}$$

Q2. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Sol. Radii of the given spheres are :

$r_1 = 6$ cm, $r_2 = 8$ cm, $r_3 = 10$ cm

$\Rightarrow$ Volume of the given spheres are :

$$V_1 = \frac{4}{3}\pi r_1^3,\ V_2 = \frac{4}{3}\pi r_2^3 \text{ and } V_3 = \frac{4}{3}\pi r_3^3$$

$\therefore$ Total volume of the given spheres = $V_1 + V_2 + V_3$

$$= \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi[r_1^3 + r_2^3 + r_3^3]$$

$$= \frac{4}{3} \times \frac{22}{7} \times [6^3 + 8^3 + 10^3]\text{cm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times [216 + 512 + 1000]\text{cm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times [1728] \text{ cm}^3$$

Let the radius of the new big sphere be R.

$\therefore$ Volume of the new sphere = $\frac{4}{3} \times \pi \times R^3$

Since, the two volume must be equal.

$$\therefore \frac{4}{3} \times \frac{22}{7} \times R^3 = \frac{4}{3} \times \frac{22}{7} \times 1728 \text{ cm}^3$$

$\Rightarrow R^3 = 1728 \Rightarrow R = 12$ cm

Thus, the required radius of the resulting sphere = 12 cm.

Q3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Sol. Diameter of the cylindrical well = 7 m

$\Rightarrow$ Radius of the cylindrical (r) = $\frac{7}{2}$ m

Depth of the well (h) = 20 m

$\therefore$ Volume = $\pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20\ m^2$

$= 22 \times 7 \times 5\ m^3$

$\Rightarrow$ Volume of the earth taken out = $22 \times 7 \times 5\ m^3$

Now this earth is spread out to form a cuboidal platform having length = 22 m, breadth = 14 m

Let h be the height of the platform.

$\therefore$ Volume of the platform = $22 \times 14 \times h\ m^3$

$\therefore$ $22 \times 14 \times h = 22 \times 7 \times 5$

$\Rightarrow h = \frac{22 \times 7 \times 5}{22 \times 14} = \frac{5}{2}$ m = 2.5 m

Thus, the required height of the platform is 2.5 m

Q4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Sol. Diameter of cylindrical well (d) = 3 m

$\Rightarrow$ Radius of the cylindrical well = $\frac{3}{2}$ m = 1.5m

Depth of the well (h) = 14 m

$\therefore$ Volume = $\pi r^2 h = \frac{22}{7} \times \left(\frac{15}{10}\right)^2 \times 14\ m^3$

$= \frac{22 \times 15 \times 15 \times 14}{7 \times 10 \times 10}\ m^3 = 99\ m^3$

Let the height of the embankment = H metre.

Internal radius of the embankment (r) = 1.5 m .

External radius of the embankment R = (4 + 1.5)m

= 5.5 m.

$\therefore$ Volume of the embankment

$= \pi R^2 H - \pi r^2 H = \pi H[R^2 - r^2] = \pi H(R + r)(R - r)$

$= \frac{22}{7} \times H(5.5 + 1.5)(5.5 - 1.5)$

$= \frac{22}{7} \times H \times 7 \times 4m^3$

Since, Volume of the embankment = Volume of the cylindrical well

$\therefore \left[\frac{22}{7} \times H \times 7 \times 4\right] = 99$

$\Rightarrow H = 99 \times \frac{7}{22} \times \frac{1}{7} \times \frac{1}{4} m = \frac{9}{8} m = 1.125 m$

$= \frac{9}{8} m = 1.125 m$

Thus, the required height of the embankment

= 1.125 m

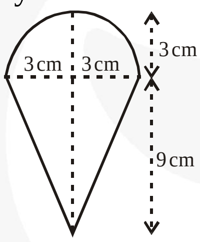
Q5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

Sol. Volume of one ice-cream cone as shown in figure.

$= \frac{1}{3}\pi \times (3)^2 \times 9 + \frac{2}{3}\pi \times (3)^3 cm^3$

$= 27\pi + 18\pi = 45\pi cm^3$

volume of the ice-cream in the cylindrical container (of height 15 cm and diameter 12 cm) $= \pi \times (6)^2 \times 15$



Let the number of cones made be n.

Then, $n \times 45\pi = \pi \times (6)^2 \times 15$

$\Rightarrow 45n = 36 \times 15 \Rightarrow n = 12$

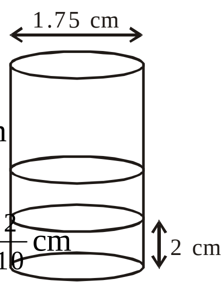
Q6. How many silver coins 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Sol. For a circular coin :

Diameter = 1.75 cm

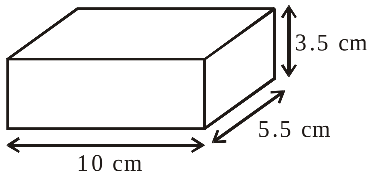
$\Rightarrow$ Radius (r) $= \frac{175}{200}$ cm

Thickness (h) = 2mm $= \frac{2}{10}$ cm



$\therefore$ Volume = $\pi r^2 h = \frac{22}{7} \times \left(\frac{175}{200}\right)^2 \times \frac{2}{10}$ cm^3

For a cuboid : Length (ℓ) = 10 cm,



Breadth (b) = 5.5 cm and Height (h) = 3.5 cm

$\therefore$ Volume = $\ell \times b \times h = 10 \times \frac{55}{10} \times \frac{35}{10}$ cm^3

Number of coins = $\frac{\text{Volume of cuboid}}{\text{Volume of one coin}}$

$$= \frac{10 \times \frac{55}{10} \times \frac{35}{10}}{\frac{22}{7} \times \left(\frac{175}{200}\right)^2 \times \frac{2}{10}} = 400$$

Thus, the required number of coins = 400.

Q7. A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Sol. For the cylindrical bucket :

Radius (r) = 18 cm and height (h) = 32 cm

Volume = $\pi r^2 h = \frac{22}{7} (18)^2 \times 32$ cm^3

$\Rightarrow$ Volume of the sand = $\left(\frac{22}{7} \times 18 \times 18 \times 32\right)$ cm^3

For the conical heap :

Height (H) = 24 cm & let radius of the base be (R).

$\therefore$ Volume of conical heap

$$= \frac{1}{3}\pi R^2 H = \left[\frac{1}{3} \times \frac{22}{7} \times R^2 \times 24\right] \text{cm}^3$$

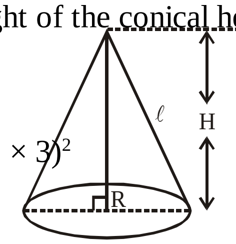
$\because$ Volume of conical heap sand = Volume of the sand

$\therefore \frac{1}{3} \times \frac{22}{7} \times R^2 \times 24 = \frac{22}{7} \times 18 \times 18 \times 32$

$\Rightarrow R^2 = \frac{18 \times 18 \times 32 \times 3}{24} = 18^2 \times 2^2$

$\Rightarrow R = \sqrt{18^2 \times 2^2} = 18 \times 2$ cm = 36 cm

Let 'ℓ' be the slant height of the conical heap of the sand.



$\therefore \quad \ell^2 = H^2 + R^2$

$\Rightarrow \quad \ell^2 = 24^2 + 36^2$

$\Rightarrow \quad \ell^2 = (12 \times 2)^2 + (12 \times 3)^2$

$\Rightarrow \quad \ell^2 = 12^2[2^2 + 3^2]$

$\Rightarrow \quad \ell^2 = 12^2 \times 13$

$\Rightarrow \quad \ell = \sqrt{12^2 \times 13} = 12 \times \sqrt{13}$

Thus, the required radius = 36 cm and slant height = $12\sqrt{13}$ cm

Q8. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Sol. Depth of water in the canal = 1.5 m

Width of canal = 6 m

Volume of water flowing through the canal in 60 minutes = (10 × 1000) × 6 × 1.5 cm^3

($\because$ Speed = 10 km/hr = 10 × 1000 m per hr)

Volume of water flowing through canal in 30 minutes

$$= 10000 \times 9 \times \frac{30}{60} m^3 = 45000 \ m^3$$

Let the required area be x m^2.

Then $x \times \frac{8}{100} = 45000$

($\because$ 8 cm deep water = $\frac{8}{100}$ m deep)

$\Rightarrow \quad x = \frac{4500000}{8} m^2 = 562500 \ m^2 = \frac{562500}{10000}$ hectares = 56.25 hectares

Q9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Sol. Water speed = 3 km/hr = $\frac{3000}{60} m/\min$

= 50 m/min

Diameter of the pipe = 20 cm

i.e., radius = 10 cm = $\frac{1}{10}$ m

Water tank has 2m depth and 10 m diameter, i.e., radius 5 m

Let the required time to fill the tank be n minutes.

Then water flowing through the pipe in n minutes = Volume of the water tank.

$$\Rightarrow \pi \times \left(\frac{1}{10}\right)^2 \times \{n \times 50\} = \pi \times (5)^2 \times 2$$

$$\frac{1}{100} \times n \times 50 = 50 \Rightarrow n = 100$$

Hence, the required time is 100 minutes.