Ex - 4.3

Q1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}\,x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Sol. (i) $2x^2 - 7x + 3 = 0$

$\Rightarrow 4x^2 - 14x + 6 = 0$

$\Rightarrow (2x)^2 - 7(2x) + 6 = 0$

$\Rightarrow y^2 - 7y + 6 = 0$ where $y = 2x$

$$\Rightarrow \left\{y^2 - \frac{7}{2}y - \frac{7}{2}y + \left(-\frac{7}{2}\right)^2\right\} - \left(-\frac{7}{2}\right)^2 + 6 = 0$$

$$\Rightarrow \left\{y - \frac{7}{2}\right\}^2 - \frac{49}{4} + 6 = 0 \Rightarrow \left\{y - \frac{7}{2}\right\}^2 - \frac{25}{4} = 0$$

$$\Rightarrow \left(2x - \frac{7}{2}\right)^2 = \frac{25}{4} \Rightarrow 2x - \frac{7}{2} = \pm\sqrt{\frac{25}{4}}$$

$$\Rightarrow 2x - \frac{7}{2} = \pm\frac{5}{2}$$

$$\Rightarrow 2x - \frac{7}{2} = \frac{5}{2} \text{ or } 2x - \frac{7}{2} = -\frac{5}{2}$$

$$\Rightarrow 2x = \frac{5}{2} + \frac{7}{2} \text{ or } 2x = -\frac{5}{2} + \frac{7}{2}$$

$\Rightarrow 2x = 6$ or $2x = 1$

Hence, the roots of the given quadratic equation are $\frac{1}{2}$ and 3.

(ii) $2x^2 + x - 4 = 0$

$\Rightarrow 2x^2 + x = 4$

On dividing both sides of the equation by 2,

we obtain $\Rightarrow x^2 + \frac{1}{2}x = 2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$$

$$\Rightarrow x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$$

$$\Rightarrow x = \frac{\pm\sqrt{33}}{4} - \frac{1}{4} \quad \Rightarrow x = \frac{\sqrt{33} - 1}{4} \text{ or } \frac{-\sqrt{33} - 1}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow (2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0$

$\Rightarrow \left(2x+\sqrt{3}\right)^2 = 0$

$\Rightarrow \left(2x+\sqrt{3}\right) = 0$ and $\left(2x+\sqrt{3}\right) = 0$

$\Rightarrow x = \frac{-\sqrt{3}}{2}$ and $x = \frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

$\Rightarrow \ 2x^2 + x = -4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow \ x^2 + \frac{1}{2}x = -2$

$\Rightarrow \ x^2 + 2 \times x \times \frac{1}{4} = -2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow \ (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2$

$\Rightarrow \ \left(x+\frac{1}{4}\right)^2 = \frac{1}{16} - 2$

$\Rightarrow \ \left(x+\frac{1}{4}\right)^2 = -\frac{31}{16}$

However, the square of a number cannot be negative. Therefore, there is no real root for the given equation.

Q2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Sol. (i) $2x^2 - 7x + 3 = 0$

$a = 2, b = -7, c = 3$

Discriminant $D = (-7)^2 - 4(2)(3) = 25$

$\Rightarrow \sqrt{D} = \sqrt{25} = 5$

Roots of the given quadratic equation are

$\frac{-b \pm \sqrt{D}}{2a}$,

i.e., $\frac{7 \pm 5}{2 \times 2}$, i.e., the roots are 3 and $\frac{1}{2}$.

(ii) $2x^2 + x - 4 = 0$

$a = 2, b = 1, c = -4$

$D = (1)^2 - 4(2)(-4) = 33 \Rightarrow \sqrt{D} = \sqrt{33}$

Roots are given by $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{33}}{4}$

Hence, the two roots of the quadratic equation are $\frac{-1 \pm \sqrt{33}}{4}$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 4$, $b = 4\sqrt{3}$, $c = 3$
By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm 0}{8}$$

$$\therefore \quad x = \frac{-\sqrt{3}}{2} \text{ or } \frac{-\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

On comparing this equation with
$ax^2 + bx + c = 0$,
we obtain $a = 2$, $b = 1$, $c = 4$
By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \quad x = \frac{-1 \pm \sqrt{1 - 32}}{4}$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{-31}}{4}$$

However, the square of a number cannot be negative. Therefore, there is no real root for the given equation.

Q3. Find the roots of the following equations :

(i) $x - \frac{1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Sol. (i) $x - \frac{1}{x} = 3 \quad \Rightarrow \quad x^2 - 3x - 1 = 0$

On comparing this equation with
$ax^2 + bx + c = 0$,
we obtain $a = 1$, $b = -3$, $c = -1$
By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \Rightarrow \quad x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$\Rightarrow \quad x = \frac{3 \pm \sqrt{13}}{2}$$

Therefore, $x = \frac{3 + \sqrt{13}}{2}$ or $\frac{3 - \sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$; x ≠ – 4 and x ≠ 7

$\Rightarrow \frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30} \Rightarrow \frac{-11}{(x^2-3x-28)} = \frac{11}{30}$

$\Rightarrow -(x^2 - 3x - 28) = 30$

$\Rightarrow x^2 - 3x - 28 + 30 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

a = 1, b = – 3, c = 2

$D = (-3)^2 - 4(1)(2) = 1 \Rightarrow \sqrt{D} = 1$

The two roots are given by $\frac{-b \pm \sqrt{D}}{2a}$, i.e., $\frac{3 \pm 1}{2}$.

Hence, the two roots are 1 and 2.

Q4. The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

Sol. Let the present age of Rehman be x years.

We are given that $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow 3(2x + 2) = (x - 3)(x + 5)$

$\Rightarrow 6x + 6 = x^2 + 2x - 15$

$\Rightarrow x^2 - 4x - 21 = 0$

a = 1, b = – 4, c = – 21

$D = (-4)^2 - 4(1)(-21) = 16 + 84 = 100$

$\Rightarrow \sqrt{D} = \sqrt{100} = 10$

Then $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{4 \pm 10}{2}$, i.e., x = 7, – 3

We reject x = – 3 ($\because$ x cannot be negative)

$\Rightarrow$ x = 7

Hence, Rehman's present age = 7 years.

Q5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Sol. Let the marks in Maths be x.

Then, the marks in English will be 30 – x.

According to the given question,

(x + 2) (30 – x – 3) = 210

(x + 2) (27 – x) = 210

$\Rightarrow$ $-x^2 + 25x + 54 = 210$

$\Rightarrow$ $x^2 - 25x + 156 = 0$

$\Rightarrow$ $x^2 - 12x - 13x + 156 = 0$

$\Rightarrow$ $x(x - 12) - 13(x - 12) = 0$

$\Rightarrow$ $(x - 12)(x - 13) = 0$

$\Rightarrow$ $x = 12, 13$

If the marks in Maths are 12, then marks in English will be 30 – 12 = 18

If the marks in Maths are 13, then marks in English will be 30 – 13 = 17.

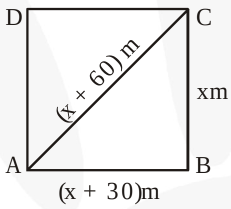
Q6. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Sol. In rectangle ABCD, let the shorter side BC = x metres.

Then AB = (x + 30) metres and diagonal

AC = (x + 60) metres.

By Pythagoras Theorem we have



$AB^2 + BC^2 = AC^2$

$\Rightarrow$ $(x + 30)^2 + x^2 = (x + 60)^2$

$\Rightarrow$ $x^2 + 60x + 900 + x^2 = x^2 + 120x + 3600$

$\Rightarrow$ $x^2 - 60x - 2700 = 0$

$a = 1,\ b = -60,\ c = -2700$

$D = (-60)^2 - 4 \times 1 \times (-2700)$

$= 3600 + 10800 = 14400$

$\Rightarrow$ $\sqrt{D} = \sqrt{14400} = 120$

Then $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{60 \pm 120}{2} = 90, -30$

We reject $x = -30$ $(\because x \nless 0)$

$\Rightarrow$ $x = 90$

$\Rightarrow$ BC = 90 m and AB = (90 + 30) m = 120 m.

Hence, the sides of the rectangular field are 90 m and 120 m.

Q7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Sol. Let the larger and smaller number be x and y respectively.
According to the given question,
$x^2 - y^2 = 180$ and $y^2 = 8x$
$\Rightarrow x^2 - 8x = 180$
$\Rightarrow x^2 - 8x - 180 = 0$
$\Rightarrow x^2 - 18x + 10x - 180 = 0$
$\Rightarrow x(x - 18) + 10(x - 18) = 0$
$\Rightarrow (x - 18)(x + 10) = 0$
$\Rightarrow x = 18, -10$
However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.
Therefore, the larger number will be 18 only.
$x = 18$
$\therefore y^2 = 8x = 8 \times 18 = 144$
$\Rightarrow y = \pm\sqrt{144} = \pm 12$
$\therefore$ Smaller number $= \pm 12$
Therefore, the numbers are 18 and 12 or 18 and – 12.

Q8. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken I hour less for the same journey. Find the speed of the train.

Sol. Let the speed of the train be x km/hr.
We are given that 360 km distance is to be travelled at uniform speed of x km/hr.
Time taken to cover the distance of
360 km $= \frac{360}{x}$ hours.
In case, the speed is increased by 5 km/hr, the time required to cover 360 km $= \frac{360}{(x+5)}$ hours.
Now, $\frac{360}{x} - \frac{360}{x+5} = 1$
$\Rightarrow 360 \times \left\{\frac{1}{x} - \frac{1}{x+5}\right\} = 1$
$\Rightarrow \frac{360 \times 5}{x \times (x+5)} = 1$
$\Rightarrow x \times (x + 5) = 360 \times 5$
$\Rightarrow x^2 + 5x = 1800 \Rightarrow x^2 + 5x - 1800 = 0$
$a = 1, b = 5, c = -1800$
$D = b^2 - 4ac = (5)^2 - 4 \times 1 \times (-1800)$
$= 25 + 7200 = 7225$
$\Rightarrow \sqrt{D} = \sqrt{7225} = 85$

Then x = $\frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm 85}{2}$

$\Rightarrow$ x = $\frac{-5 \pm 85}{2}$ or $\frac{-5-85}{2}$

$\Rightarrow$ x = 40 or – 45 We reject x = – 45

$\Rightarrow$ x = 40

Q9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol. Let the time taken by the smaller pipe to fill the tank be x hr.
Time taken by the larger pipe = (x - 10) hr

Part of tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together.

Therefore,

$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$

$\frac{x-10+x}{x(x-10)} = \frac{8}{75}$

$\Rightarrow$ $75(2x - 10) = 8x^2 - 80x$

$\Rightarrow$ $150x - 750 = 8x^2 - 80x$

$\Rightarrow$ $8x^2 - 230x + 750 = 0$

$\Rightarrow$ $8x^2 - 200x - 30x + 750 = 0$

$\Rightarrow$ $8x(x - 25) - 30(x - 25) = 0$

$\Rightarrow$ $(x - 25)(8x - 30) = 0$

i.e., x = 25, $\frac{30}{8}$

Time taken by the smaller pipe cannot be $\frac{30}{8}$ = 3.75 hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible.
Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 – 10 = 15 hours respectively.

Q10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop in intermediate stations). If the average speed of the express train is 11 km/hr more than that of the passenger train, find the average speed of the two trains.

Sol. Let the average speed of the passenger train be x km/hr.
Then the average speed of express train = (x + 11) km/hr.
According to the given condition :

$$\begin{pmatrix}\text{Time taken by passenger train} \\ \text{for 132 km journey}\end{pmatrix}$$

$$\begin{pmatrix}\text{Time taken by the express train} \\ \text{for 132 km journey}\end{pmatrix} = 1 \text{ hour}$$

$$\Rightarrow \left(\frac{132}{x} - \frac{132}{x+11}\right) = 1$$

$$\Rightarrow 132 \times \left\{\frac{1}{x} - \frac{1}{x+11}\right\} = 1$$

$$\Rightarrow 132 \times \frac{11}{x \times (x+11)} = 1$$

$\Rightarrow 132 \times 11 = x \times (x + 11)$

$\Rightarrow 1452 = x^2 + 11x \Rightarrow x^2 + 11x - 1452 = 0$

$\Rightarrow a = 1, b = 11, c = -1452$

$D = b^2 - 4ac = 121 - 4 \times 1 \times (-1452)$

$= 121 + 5808 = 5929$

$$\Rightarrow \sqrt{D} = \sqrt{5929} = 77$$

$$\text{Then } x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-11 \pm 77}{2} = -44 \text{ or } 33$$

We reject $x = -44 \Rightarrow x = 33$

Hence, the speed of passenger train = 33 km/hr and the speed of express train = 44 km/hr

Q11. Sum of the area of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Sol. Let the sides of the two squares be x m and y m. Therefore, their perimeter will be 4x and 4y respectively and their areas will be x^2 and y^2 respectively.

It is given that $4x - 4y = 24$

$x - y = 6$

$x = y + 6$

Also, $x^2 + y^2 = 468$

$\Rightarrow (6 + y)^2 + y^2 = 468$

$\Rightarrow 36 + y^2 + 12y + y^2 = 468$

$\Rightarrow 2y^2 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y + 18)(y - 12) = 0$

$\Rightarrow y = -18$ or 12.

However, side of a square cannot be negative.

Hence, the sides of the squares are 12 m and

$(12 + 6)$ m = 18 m