Ex - 7.1

Q1. Find the distance between the following pairs of points :
(a) (2,3), (4, 1)
(b) (–5, 7), (–1,3)
(c) (a, b), (– a, – b)

Sol.(a) The given points are : A (2, 3), B (4, 1).

Required distance = AB = BA = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AB = $\sqrt{(4-2)^2 + (1-3)^2} = \sqrt{(2)^2 + (-2)^2}$

= $\sqrt{4+4}$ = $\sqrt{8}$ = $2\sqrt{2}$ units

(b) Here $x_1 = -5$, $y_1 = 7$ and $x_2 = -1$, $y_2 = 3$

∴ The required distance

= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{[-1-(-5)]^2 + (3-7)^2}$

= $\sqrt{(-1+5)^2 + (-4)^2}$

= $\sqrt{16+16} = \sqrt{32} = \sqrt{2 \times 16}$

= $4\sqrt{2}$ units

(c) Here $x_1 = a$, $y_1 = b$ and $x_2 = -a$, $y_2 = -b$

∴ The required distance

= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(-a-a)^2 + (-b-b)^2}$

= $\sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2}$

= $\sqrt{4(a^2 + b^2)} = 2\sqrt{(a^2 + b^2)}$ units

Q2. Find the distance between the points (0,0) and (36,15).

Sol. **Part-I**

Let the points be A(0, 0) and B(36, 15)

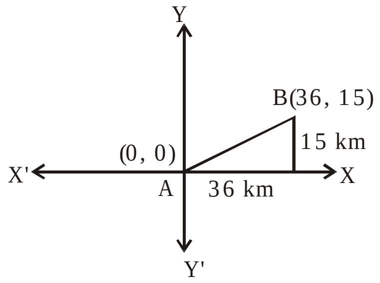
∴ AB = $\sqrt{(36-0)^2 + (15-0)^2}$

= $\sqrt{(36)^2 + (15)^2}$ = $\sqrt{1296 + 225}$

= $\sqrt{1521} = \sqrt{39^2}$ = 39

Part-II

We have A(0, 0) and B(36, 15) as the positions of two towns



Here $x_1 = 0$, $x_2 = 36$ and $y_1 = 0$, $y_2 = 15$

$\therefore$ $AB = \sqrt{(36-0)^2 + (15-0)^2} = 39$ km

Q3. Determine if the points (1,5), (2,3) and (– 2, – 11) are collinear.

Sol. The given points are :

A(1, 5), B(2, 3) and C(–2, –11).

Let us calculate the distance : AB, BC and CA by using distance formula.

$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2}$

$= \sqrt{1+4} = \sqrt{5}$ units

$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2}$

$= \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$ units

$CA = \sqrt{(-2-1)^2 + (-11-5)^2}$

$= \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$

$= \sqrt{5} \times \sqrt{53}$ units

From the above we see that : $AB + BC \neq CA$

Hence the above stated points A(1, 5), B(2, 3) and C(–2, –11) are not collinear.

Q4. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

Sol. Let the points be A(5, –2), B(6, 4) and C(7, –2).

$\therefore$ $AB = \sqrt{(6-5)^2 + [4-(-2)]^2}$

$= \sqrt{(1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$

$BC = \sqrt{(7-6)^2 + (-2-4)^2}$

$= \sqrt{(1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$

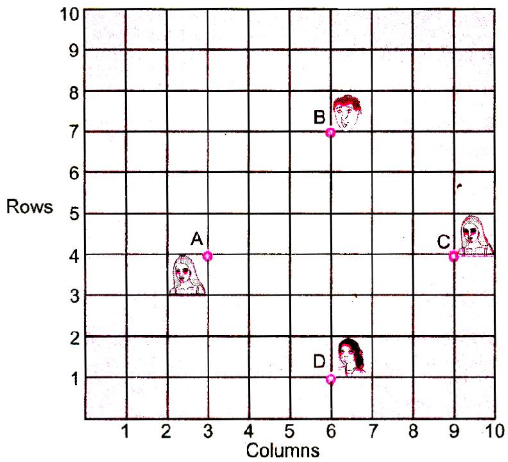
$AC = \sqrt{(7-5)^2 + (-2-(-2))^2}$

$= \sqrt{(+2)^2 + (0)^2} = \sqrt{4+0} = 2$

We have $AB = BC \neq AC$.

$\therefore$ ΔABC is an isosceles triangle.

Q5. In a classroom, 4 friends are seated at the points A, B, C and D as shown in fig. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a rectangle?" Chameli disagrees. Using distance formula, find which of them is correct.



Sol. Let the number of horizontal columns represent the x-coordinates whereas the vertical rows represent the y-coordinates.

$\therefore$ The points are : A(3, 4), B(6, 7), C(9, 4) and D(6, 1)

$\therefore$ $AB = \sqrt{(6-3)^2 + (7-4)^2}$

$= \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(9-6)^2 + (4-7)^2}$

$= \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(6-9)^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$AD = \sqrt{(6-3)^2 + (1-4)^2}$

$= \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Since, AB = BC = CD = AD i.e., All the four sides are equal

Also $AC = \sqrt{(9-3)^2 + (4-4)^2}$

$= \sqrt{(+6)^2 + (0)^2} = 6$ and

$BD = \sqrt{(6-6)^2 + (1-7)^2} = \sqrt{(0)^2 + (-6)^2} = 6$

i.e., BD = AC

$\Rightarrow$ Both the diagonals are also equal.

$\therefore$ ABCD is a square.

Thus, Chameli is correct as ABCD is not a rectangle.

Q6. Name the quadrilateral formed, if any, by the following points, and give reasons for your answer.

(i) (–1, –2), (1, 0), (–1, 2), (– 3, 0)

(ii) (–3, 5), (3, 1), (0, 3), (–1, – 4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Sol. (i) A(–1, –2), B(1, 0), C(–1, 2), D(–3, 0)

Determine distances : AB, BC, CD, DA, AC and BD.

$$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$DA = \sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

AB = BC = CD = DA

The sides of the quadrilateral are equal ....(1)

$$\left.\begin{aligned} AC &= \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = 4 \\ BD &= \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = 4 \end{aligned}\right\}$$

Diagonal AC = Diagonal BD..........(2)

From (1) and (2) we conclude that ABCD is a square.

(ii) Let the points be A(–3, 5), B(3, 1), C(0, 3) and D(–1, –4).

$\therefore$ AB = $\sqrt{[3-(-3)]^2 + (1-5)^2}$

$= \sqrt{6^2 + (-4)^2} = \sqrt{36+16}$

$= \sqrt{52} = 2\sqrt{13}$

BC = $\sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$

CD = $\sqrt{(-1-0)^2 + (-4-3)^2} = \sqrt{(-1)^2 + (-7)^2}$

$= \sqrt{1+49} = \sqrt{50}$

DA = $\sqrt{[-3-(-1)]^2 + [5-(-4)]^2}$

$= \sqrt{(-2)^2 + (9)^2}$

$= \sqrt{4+81} = \sqrt{85}$

AC = $\sqrt{[0-(-3)]^2 + (3-5)^2} = \sqrt{(3)^2 + (-2)^2}$

$= \sqrt{9+4} = \sqrt{13}$

BD = $\sqrt{(-1-3)^2 + (-4-1)^2} = \sqrt{(-4)^2 + (-5)^2}$

$= \sqrt{16+25} = \sqrt{41}$

We see that $\sqrt{13} + \sqrt{13} = 2\sqrt{13}$

i.e., AC + BC = AB

$\Rightarrow$ A, B and C are collinear. Thus, ABCD is not a quadrilateral.

(iii) Let the points be A(4, 5), B(7, 6), C(4, 3) and D(1, 2).

$\therefore$ AB = $\sqrt{(7-4)^2+(6-5)^2} = \sqrt{3^2+1^2} = \sqrt{10}$

BC = $\sqrt{(4-7)^2+(3-6)^2}$

$= \sqrt{(-3)^2+(-3)^2} = \sqrt{18}$

CD = $\sqrt{(1-4)^2+(2-3)^2}$

$= \sqrt{(-3)^2+(-1)^2} = \sqrt{10}$

DA = $\sqrt{(1-4)^2+(2-5)^2} = \sqrt{9+9} = \sqrt{18}$

AC = $\sqrt{(4-4)^2+(3-5)^2} = \sqrt{0+(-2)^2} = 2$

BD = $\sqrt{(1-7)^2+(2-6)^2} = \sqrt{36+16} = \sqrt{52}$

Since, AB = CD, BC = DA [opposite sides of the quadrilateral are equal]

And AC $\neq$ BD $\Rightarrow$ Diagonals are unequal.

$\therefore$ ABCD is a parallelogram.

Q7. Find the point on the x-axis which is equidistant from (2, – 5) and (– 2, 9).

Sol. We know that any point on x-axis has its ordinate = 0

Let the required point be P(x, 0).

Let the given points be A(2, –5) and B(–2, 9)

$\therefore$ AP = $\sqrt{(x-2)^2+5^2} = \sqrt{x^2-4x+4+25}$

$= \sqrt{x^2-4x+29}$

BP = $\sqrt{[x-(-2)]^2+(-9)^2} = \sqrt{(x+2)^2+(-9)^2}$

$= \sqrt{x^2+4x+4+81} = \sqrt{x^2+4x+85}$

Since, A and B are equidistant from P,

$\therefore$ AP = BP

$\Rightarrow \sqrt{x^2-4x+29} = \sqrt{x^2+4x+85}$

$\Rightarrow x^2 - 4x + 29 = x^2 + 4x + 85$

$\Rightarrow x^2 - 4x - x^2 - 4x = 85 - 29$

$\Rightarrow -8x = 56 \Rightarrow x = \frac{56}{-8} = -7$

$\therefore$ The required point is (–7, 0)

Q8. Find the values of y for which the distance between the points P(2, –3) and Q(10, y) is 10 units.

Sol. Distance between P(2, – 3) and Q(10, y) = 10 units

$\Rightarrow \sqrt{(10-2)^2+(y+3)^2}=10$

$\Rightarrow 64 + (y + 3)^2 = 100$

$\Rightarrow (y + 3)^2 = 36$

$\Rightarrow y^2 + 6y + 9 = 36$

$y^2 + 6y - 27 = 0$

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y + 9) - 3(y + 9) = 0$

$\Rightarrow (y + 9)(y - 3) = 0$

$\Rightarrow y + 9 = 0$ or $y - 3 = 0$

$\Rightarrow y = -9$ or 3

Hence, there can be two values of y which are – 9 and 3.

Q9. If Q (0,1) is equidistant from P(5, – 3) and R(x, 6), find the values of x. Also find the distances QR and PR.

Sol. Here, QP = $\sqrt{(5-0)^2+[(-3)-1]^2}=\sqrt{5^2+(-4)^2}$

$=\sqrt{25+16}=\sqrt{41}$

QR = $\sqrt{(x-0)^2+(6-1)^2}=\sqrt{x^2+5^2}=\sqrt{x^2+25}$

$\because$ QP = QR

$\therefore \sqrt{41}=\sqrt{x^2+25}$

Squaring both sides, we have $x^2 + 25 = 41$

$\Rightarrow x^2 + 25 - 41 = 0$

$\Rightarrow x^2 - 16 = 0 \Rightarrow x = \pm\sqrt{16}=\pm 4$

Thus, the point R is (4, 6) or (–4, 6)

Now,

QR = $\sqrt{[(\pm 4)-(0)]^2+(6-1)^2}=\sqrt{16+25}=\sqrt{41}$

and PR = $\sqrt{(\pm 4-5)^2+(6+3)^2}$

$\Rightarrow$ PR = $\sqrt{(-4-5)^2+(6+3)^2}$

or $\sqrt{(4-5)^2+(6+3)^2}$

$\Rightarrow$ PR = $\sqrt{(-9)^2+9^2}$ or $\sqrt{1+81}$

$\Rightarrow$ PR = $\sqrt{2\times 9^2}$ or $\sqrt{82}$

$\Rightarrow$ PR = $9\sqrt{2}$ or $\sqrt{82}$

Q10. Find a relation between x and y such that the point (x,y) is equidistant from the point (3, 6) and (– 3, 4).

Sol. A(3,6) and B(–3, 4) are the given points. Point P (x, y) is equidistant from the points A and B.

$\Rightarrow$ PA = PB

$\Rightarrow \sqrt{(x-3)^2+(y-6)^2} = \sqrt{(x+3)^2+(y-4)^2}$

$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$\Rightarrow (x^2 - 6x + 9) + (y^2 - 12y + 36)$

$= (x^2 + 6x + 9) + (y^2 - 8y + 16)$

$\Rightarrow - 6x - 12y + 45 = 6x - 8y + 25$

$\Rightarrow 12x + 4y - 20 = 0 \Rightarrow 3x + y - 5 = 0$