

Exercise 5.2**Question 1:**

Find the modulus and the argument of the complex number $z = -1 - i\sqrt{3}$

Answer

$$z = -1 - i\sqrt{3}$$

$$\text{Let } r \cos \theta = -1 \text{ and } r \sin \theta = -\sqrt{3}$$

On squaring and adding, we obtain

$$(r \cos \theta)^2 + (r \sin \theta)^2 = (-1)^2 + (-\sqrt{3})^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 3$$

$$\Rightarrow r^2 = 4 \quad [\cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow r = \sqrt{4} = 2 \quad [\text{Conventionally, } r > 0]$$

$$\therefore \text{Modulus} = 2$$

$$\therefore 2 \cos \theta = -1 \text{ and } 2 \sin \theta = -\sqrt{3}$$

$$\Rightarrow \cos \theta = \frac{-1}{2} \text{ and } \sin \theta = \frac{-\sqrt{3}}{2}$$

Since both the values of $\sin \theta$ and $\cos \theta$ are negative and $\sin \theta$ and $\cos \theta$ are negative in III quadrant,

$$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$$

Thus, the modulus and argument of the complex number $-1 - \sqrt{3}i$ are 2 and $\frac{-2\pi}{3}$ respectively.

Question 2:

Find the modulus and the argument of the complex number $z = -\sqrt{3} + i$

Answer

$$z = -\sqrt{3} + i$$

$$\text{Let } r \cos \theta = -\sqrt{3} \text{ and } r \sin \theta = 1$$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-\sqrt{3})^2 + 1^2$$

$$\Rightarrow r^2 = 3 + 1 = 4$$

$$[\cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow r = \sqrt{4} = 2$$

$$[\text{Conventionally, } r > 0]$$

$$\therefore \text{Modulus} = 2$$

$$\therefore 2 \cos \theta = -\sqrt{3} \text{ and } 2 \sin \theta = 1$$

$$\Rightarrow \cos \theta = \frac{-\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2}$$

$$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$[\text{As } \theta \text{ lies in the II quadrant}]$$

Thus, the modulus and argument of the complex number $-\sqrt{3} + i$ are 2 and $\frac{5\pi}{6}$ respectively.

Question 3:

Convert the given complex number in polar form: $1 - i$

Answer

$$1 - i$$

$$\text{Let } r \cos \theta = 1 \text{ and } r \sin \theta = -1$$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 1^2 + (-1)^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$$

$$\Rightarrow r^2 = 2$$

$$\Rightarrow r = \sqrt{2} \quad [\text{Conventionally, } r > 0]$$

$$\therefore \sqrt{2} \cos \theta = 1 \text{ and } \sqrt{2} \sin \theta = -1$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\frac{\pi}{4} \quad [\text{As } \theta \text{ lies in the IV quadrant}]$$

$$\therefore 1 - i = r \cos \theta + i r \sin \theta = \sqrt{2} \cos \left(-\frac{\pi}{4} \right) + i \sqrt{2} \sin \left(-\frac{\pi}{4} \right) = \sqrt{2} \left[\cos \left(-\frac{\pi}{4} \right) + i \sin \left(-\frac{\pi}{4} \right) \right] \text{ This is}$$

the required polar form.

Question 4:

Convert the given complex number in polar form: $-1 + i$

Answer

$$-1 + i$$

$$\text{Let } r \cos \theta = -1 \text{ and } r \sin \theta = 1$$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-1)^2 + 1^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$$

$$\Rightarrow r^2 = 2$$

$$\Rightarrow r = \sqrt{2} \quad [\text{Conventionally, } r > 0]$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1$$

$$\Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad [\text{As } \theta \text{ lies in the II quadrant}]$$

It can be written,

$$\therefore -1+i = r \cos \theta + i r \sin \theta = \sqrt{2} \cos \frac{3\pi}{4} + i \sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

This is the required polar form.

Question 5:

Convert the given complex number in polar form: $-1 - i$

Answer

$$-1 - i$$

Let $r \cos \theta = -1$ and $r \sin \theta = -1$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-1)^2 + (-1)^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$$

$$\Rightarrow r^2 = 2$$

$$\Rightarrow r = \sqrt{2} \quad [\text{Conventionally, } r > 0]$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = -1$$

$$\Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \quad [\text{As } \theta \text{ lies in the III quadrant}]$$

$$\therefore -1 - i = r \cos \theta + i r \sin \theta = \sqrt{2} \cos \frac{-3\pi}{4} + i \sqrt{2} \sin \frac{-3\pi}{4} = \sqrt{2} \left(\cos \frac{-3\pi}{4} + i \sin \frac{-3\pi}{4} \right)$$

This is the required polar form.

Question 6:

Convert the given complex number in polar form: -3

Answer

$$-3$$

Let $r \cos \theta = -3$ and $r \sin \theta = 0$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-3)^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 9$$

$$\Rightarrow r^2 = 9$$

$$\Rightarrow r = \sqrt{9} = 3 \quad [\text{Conventionally, } r > 0]$$

$$\therefore 3 \cos \theta = -3 \text{ and } 3 \sin \theta = 0$$

$$\Rightarrow \cos \theta = -1 \text{ and } \sin \theta = 0$$

$$\therefore \theta = \pi$$

$$\therefore -3 = r \cos \theta + i r \sin \theta = 3 \cos \pi + i 3 \sin \pi = 3 (\cos \pi + i \sin \pi)$$

This is the required polar form.

Question 7:

Convert the given complex number in polar form: $\sqrt{3} + i$

Answer

$$\sqrt{3} + i$$

$$\text{Let } r \cos \theta = \sqrt{3} \text{ and } r \sin \theta = 1$$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (\sqrt{3})^2 + 1^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 3 + 1$$

$$\Rightarrow r^2 = 4$$

$$\Rightarrow r = \sqrt{4} = 2 \quad [\text{Conventionally, } r > 0]$$

$$\therefore 2 \cos \theta = \sqrt{3} \text{ and } 2 \sin \theta = 1$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{6} \quad [\text{As } \theta \text{ lies in the I quadrant}]$$

$$\therefore \sqrt{3} + i = r \cos \theta + i r \sin \theta = 2 \cos \frac{\pi}{6} + i 2 \sin \frac{\pi}{6} = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

This is the required polar form.

Question 8:

Convert the given complex number in polar form: i

Answer

i

Let $r \cos \theta = 0$ and $r \sin \theta = 1$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 0^2 + 1^2$$

$$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1$$

$$\Rightarrow r^2 = 1$$

$$\Rightarrow r = \sqrt{1} = 1 \quad [\text{Conventionally, } r > 0]$$

$$\therefore \cos \theta = 0 \text{ and } \sin \theta = 1$$

$$\therefore \theta = \frac{\pi}{2}$$

$$\therefore i = r \cos \theta + i r \sin \theta = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$$

This is the required polar form.