Exercise 1.5

Question 1:

Let U ={1, 2, 3; 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find

(i) A'

(ii) B'

(iii) $(A \cup C)'$

(iv) $(A \cup B)'$

(v) $(A')'$

(vi) $(B - C)'$

Answer

U ={1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {1, 2, 3, 4}

B = {2, 4, 6, 8}

C = {3, 4, 5, 6}

(i) $A' = \{5, 6, 7, 8, 9\}$

(ii) $B' = \{1, 3, 5, 7, 9\}$

(iii) $A \cup C = \{1, 2, 3, 4, 5, 6\}$

$\therefore (A \cup C)' = \{7, 8, 9\}$

(iv) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B)' = \{5, 7, 9\}$

(v) $(A')' = A = \{1, 2, 3, 4\}$

(vi) $B - C = \{2, 8\}$

$\therefore (B - C)' = \{1, 3, 4, 5, 6, 7, 9\}$

Question 2:

If U = {*a, b, c, d, e, f, g, h*}, find the complements of the following sets:

(i) A = {*a, b, c*}

(ii) B = {*d, e, f, g*}

(iii) C = {*a, c, e, g*}

(iv) D = {*f*, *g*, *h*, *a*}

Answer

U = {*a, b, c, d, e, f, g, h*}

(i) A = {*a, b, c*}

$A' = \{d, e, f, g, h\}$

(ii) B = {*d, e, f, g*}

$\therefore B' = \{a, b, c, h\}$

(iii) C = {*a, c, e, g*}

$\therefore C' = \{b, d, f, h\}$

(iv) D = {*f*, *g*, *h*, *a*}

$\therefore D' = \{b, c, d, e\}$

Question 3:

Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) {*x*: *x* is an even natural number}

(ii) {*x*: *x* is an odd natural number}

(iii) {*x*: *x* is a positive multiple of 3}

(iv) {*x*: *x* is a prime number}

(v) {*x*: *x* is a natural number divisible by 3 and 5}

(vi) {*x*: *x* is a perfect square}

(vii) {*x*: *x* is perfect cube}

(viii) {*x*: *x* + 5 = 8}

(ix) {*x*: 2*x* + 5 = 9}

(x) {*x*: *x* ≥ 7}

(xi) {*x*: *x* ∈ N and 2*x* + 1 > 10}

Answer

U = N: Set of natural numbers

(i) {*x*: *x* is an even natural number}′ = {*x*: *x* is an odd natural number}

(ii) $\{x: x$ is an odd natural number$\}' = \{x: x$ is an even natural number$\}$

(iii) $\{x: x$ is a positive multiple of 3$\}' = \{x: x \in N$ and x is not a multiple of 3$\}$

(iv) $\{x: x$ is a prime number$\}' = \{x: x$ is a positive composite number and $x = 1\}$

(v) $\{x: x$ is a natural number divisible by 3 and 5$\}' = \{x: x$ is a natural number that is not divisible by 3 or 5$\}$

(vi) $\{x: x$ is a perfect square$\}' = \{x: x \in N$ and x is not a perfect square$\}$

(vii) $\{x: x$ is a perfect cube$\}' = \{x: x \in N$ and x is not a perfect cube$\}$

(viii) $\{x: x + 5 = 8\}' = \{x: x \in N$ and $x \neq 3\}$

(ix) $\{x: 2x + 5 = 9\}' = \{x: x \in N$ and $x \neq 2\}$

(x) $\{x: x \geq 7\}' = \{x: x \in N$ and $x < 7\}$

(xi) $\{x: x \in N$ and $2x + 1 > 10\}' = \{x: x \in N$ and $x \leq 9/2\}$

Question 4:

If U = {1, 2, 3, 4, 5,6,7,8, 9}, A = {2, 4, 6, 8} and B = {2, 3, 5, 7}. Verify that

(i) $(A \cup B)' = A' \cap B'$ **(ii)** $(A \cap B)' = A' \cup B'$

Answer

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {2, 4, 6, 8}, B = {2, 3, 5, 7}

(i)

$$(A \cup B)' = \{2, 3, 4, 5, 6, 7, 8\}' = \{1, 9\}$$
$$A' \cap B' = \{1, 3, 5, 7, 9\} \cap (1, 4, 6, 8, 9) = \{1, 9\}$$
$$\therefore (A \cup B)' = A' \cap B'$$

(ii)

$$(A \cap B)' = \{2\}' = \{1, 3, 4, 5, 6, 7, 8, 9\}$$
$$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\} = \{1, 3, 4, 5, 6, 7, 8, 9\}$$
$$\therefore (A \cap B)' = A' \cup B'$$

Question 5:

Draw appropriate Venn diagram for each of the following:

(i) $(A \cup B)'$

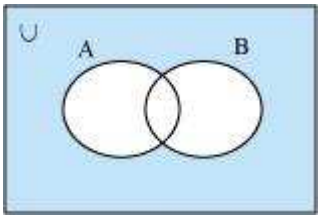
(ii) $A' \cap B'$

(iii) $(A \cap B)'$

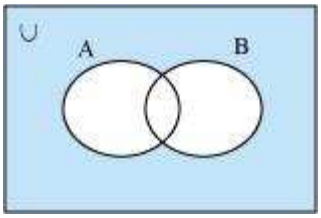
(iv) $A' \cup B'$

Answer

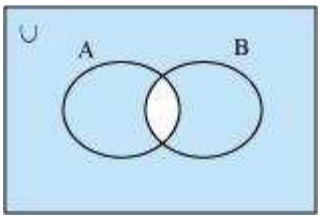
(i) $(A \cup B)'$



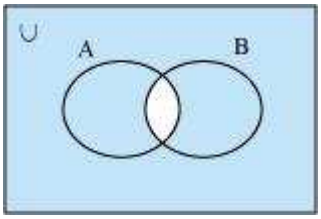
(ii) $A' \cap B'$



(iii) $(A \cap B)'$



(iv) $A' \cup B'$



Question 6:

Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A'?

Answer

A' is the set of all equilateral triangles.

Question 7:

Fill in the blanks to make each of the following a true statement:

(i) $A \cup A' = \ldots$

(ii) Φ′ ∩ A = …

(iii) $A \cap A' = \ldots$

(iv) $U' \cap A = \ldots$

Answer

(i) $A \cup A' = U$

(ii) Φ′ ∩ A = U ∩ A = A

∴ Φ′ ∩ A = A

(iii) A ∩ A′ = Φ

(iv) U′ ∩ A = Φ ∩ A = Φ

∴ U′ ∩ A = Φ

Exercise 1.6

Question 1:

If X and Y are two sets such that n(X) = 17, n(Y) = 23 and n(X ∪ Y) = 38, find n(X ∩Y).

Answer