

NCERT Miscellaneous Solutions**Question 1:**

Write the negation of the following statements:

(i) p : For every positive real number x , the number $x - 1$ is also positive.

(ii) q : All cats scratch.

(iii) r : For every real number x , either $x > 1$ or $x < 1$.

(iv) s : There exists a number x such that $0 < x < 1$.

Answer

(i) The negation of statement p is as follows.

There exists a positive real number x , such that $x - 1$ is not positive.

(ii) The negation of statement q is as follows.

There exists a cat that does not scratch.

(iii) The negation of statement r is as follows.

There exists a real number x , such that neither $x > 1$ nor $x < 1$.

(iv) The negation of statement s is as follows.

There does not exist a number x , such that $0 < x < 1$.

Question 2:

State the converse and contrapositive of each of the following statements:

(i) p : A positive integer is prime only if it has no divisors other than 1 and itself.

(ii) q : I go to a beach whenever it is a sunny day.

(iii) r : If it is hot outside, then you feel thirsty.

Answer

(i) Statement p can be written as follows.

If a positive integer is prime, then it has no divisors other than 1 and itself.

The converse of the statement is as follows.

If a positive integer has no divisors other than 1 and itself, then it is prime.

The contrapositive of the statement is as follows.

If positive integer has divisors other than 1 and itself, then it is not prime.

(ii) The given statement can be written as follows.

If it is a sunny day, then I go to a beach.

The converse of the statement is as follows.

If I go to a beach, then it is a sunny day.

The contrapositive of the statement is as follows.

If I do not go to a beach, then it is not a sunny day.

(iii) The converse of statement r is as follows.

If you feel thirsty, then it is hot outside.

The contrapositive of statement r is as follows.

If you do not feel thirsty, then it is not hot outside.

Question 3:

Write each of the statements in the form “if p , then q ”.

(i) p : It is necessary to have a password to log on to the server.

(ii) q : There is traffic jam whenever it rains.

(iii) r : You can access the website only if you pay a subscription fee.

Answer

(i) Statement p can be written as follows.

If you log on to the server, then you have a password.

(ii) Statement q can be written as follows.

If it rains, then there is a traffic jam.

(iii) Statement r can be written as follows.

If you can access the website, then you pay a subscription fee.

Question 4:

Re write each of the following statements in the form “ p if and only if q ”.

(i) p : If you watch television, then your mind is free and if your mind is free, then you watch television.

(ii) q : For you to get an A grade, it is necessary and sufficient that you do all the homework regularly.

(iii) r : If a quadrilateral is equiangular, then it is a rectangle and if a quadrilateral is a rectangle, then it is equiangular.

Answer

(i) You watch television if and only if your mind is free.

(ii) You get an A grade if and only if you do all the homework regularly.

(iii) A quadrilateral is equiangular if and only if it is a rectangle.

Question 5:

Given below are two statements

p : 25 is a multiple of 5.

q : 25 is a multiple of 8.

Write the compound statements connecting these two statements with "And" and "Or".

In both cases check the validity of the compound statement.

Answer

The compound statement with 'And' is "25 is a multiple of 5 and 8".

This is a false statement, since 25 is not a multiple of 8.

The compound statement with 'Or' is "25 is a multiple of 5 or 8".

This is a true statement, since 25 is not a multiple of 8 but it is a multiple of 5.

Question 6:

Check the validity of the statements given below by the method given against it.

(i) p : The sum of an irrational number and a rational number is irrational (by contradiction method).

(ii) q : If n is a real number with $n > 3$, then $n^2 > 9$ (by contradiction method).

Answer

(i) The given statement is as follows.

p : the sum of an irrational number and a rational number is irrational.

Let us assume that the given statement, p , is false. That is, we assume that the sum of an irrational number and a rational number is rational.

Therefore, $\sqrt{a} + \frac{b}{c} = \frac{d}{e}$, where $\sqrt{a}$ is irrational and b, c, d, e are integers.

$\frac{d}{e} - \frac{b}{c}$ is a rational number and $\sqrt{a}$ is an irrational number.

This is a contradiction. Therefore, our assumption is wrong.

Therefore, the sum of an irrational number and a rational number is irrational.

Thus, the given statement is true.

(ii) The given statement, q , is as follows.

If n is a real number with $n > 3$, then $n^2 > 9$.

Let us assume that n is a real number with $n > 3$, but $n^2 > 9$ is not true.

That is, $n^2 < 9$

Then, $n > 3$ and n is a real number.

Squaring both the sides, we obtain

$$n^2 > (3)^2$$

$\Rightarrow n^2 > 9$, which is a contradiction, since we have assumed that $n^2 < 9$.

Thus, the given statement is true. That is, if n is a real number with $n > 3$, then $n^2 > 9$.

Question 7:

Write the following statement in five different ways, conveying the same meaning.

p: If triangle is equiangular, then it is an obtuse angled triangle.

Answer

The given statement can be written in five different ways as follows.

- (i) A triangle is equiangular implies that it is an obtuse-angled triangle.
- (ii) A triangle is equiangular only if it is an obtuse-angled triangle.
- (iii) For a triangle to be equiangular, it is necessary that the triangle is an obtuse-angled triangle.
- (iv) For a triangle to be an obtuse-angled triangle, it is sufficient that the triangle is equiangular.
- (v) If a triangle is not an obtuse-angled triangle, then the triangle is not equiangular.