

Ex - 5.3

Q1. Find the sum of the following APs :

- (i) 2, 7, 12, to 10 terms.
- (ii) $-37, -33, -29, \dots$ to 12 terms.
- (iii) 0.6, 1.7, 2.8, ... to 100 terms
- (iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$ to 11 terms

Sol. (i) $a = 2, d = 5$

$$S_{10} = \frac{10}{2} \{2a + 9d\}$$

$$\left(\because S_n = \frac{n}{2} \{2a + (n-1)d\} \right)$$

$$= 5 \times \{2 \times 2 + 9 \times 5\} = 5 \times 49 = 245$$

(ii) $a = -37, d = 4$

$$S_{12} = \frac{12}{2} \{2a + 11d\}$$

$$= 6 \times \{2(-37) + 11 \times 4\}$$

$$= 6 \times \{-74 + 44\} = -180$$

(iii) 0.6, 1.7, 2.8,, to 100 terms

For this A.P.,

$$a = 0.6$$

$$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$$

$$n = 100$$

$$S_{100} = \frac{100}{2} [2(0.6) + (100-1)1.1]$$

$$= 50[1.2 + (99) \times (1.1)]$$

$$= 50[110.1]$$

$$= 110.1$$

$$= 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$, to 11 terms

For this A.P.,

$$a = \frac{1}{15}$$

$$n = 11$$

$$d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$$

We know that,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned} S_{11} &= \frac{11}{2} \left[2 \left(\frac{1}{15} \right) + (11-1) \frac{1}{60} \right] \\ &= \frac{11}{2} \left[\frac{2}{15} + \frac{10}{60} \right] \\ &= \frac{11}{2} \left[\frac{2}{15} + \frac{1}{6} \right] = \frac{11}{2} \left[\frac{4+5}{30} \right] \\ &= \left(\frac{11}{2} \right) \left(\frac{9}{30} \right) = \frac{33}{20} \end{aligned}$$

Q2. Find the sums given below :

(i) $7 + 10\frac{1}{2} + 14 + \dots + 84.$

(ii) $34 + 32 + 30 + \dots + 10$

(iii) $-5 + (-8) + (-11) + \dots + (-230).$

Sol. (i) $a = 7, d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}$
 $\ell = t_n = 84 \Rightarrow a + (n-1)d = 84$

$$\Rightarrow 7 + (n-1) \times \frac{7}{2} = 84$$

$$\Rightarrow (n-1) \times \frac{7}{2} = 77$$

$$\Rightarrow n-1 = 77 \times \frac{2}{7} = 22$$

$$\Rightarrow n = 23$$

$$\begin{aligned} \text{The sum} &= \frac{n}{2} \{a + t_n\} = \frac{23}{2} \{7 + 84\} \\ &= \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2} \end{aligned}$$

(ii) $34 + 32 + 30 + \dots + 10$

$$a = 34$$

$$d = a_2 - a_1 = 32 - 34 = -2$$

$$\ell = 10$$

Let 10 be the n th term of this A.P.

$$\ell = a + (n-1)d$$

$$10 = 34 + (n-1)(-2)$$

$$-24 = (n-1)(-2)$$

$$12 = n - 1$$

$$n = 13$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$= \frac{13}{2}(34 + 10)$$

$$= \frac{13 \times 44}{2} = 13 \times 22 = 286$$

$$(iii) (-5) + (-8) + (-11) + \dots + (-230)$$

For this A.P.,

$$a = -5$$

$$\ell = -230$$

$$d = a_2 - a_1 = (-8) - (-5)$$

$$= -8 + 5 = -3$$

Let -230 be the n^{th} term of this A.P.

$$\ell = a + (n - 1)d$$

$$-230 = -5 + (n - 1)(-3)$$

$$-225 = (n - 1)(-3)$$

$$(n - 1) = 75$$

$$n = 76$$

$$\text{And, } S_n = \frac{n}{2}(a + \ell)$$

$$= \frac{76}{2}[(-5) + (-230)]$$

$$= 38(-235)$$

$$= -8930$$

Q3. In an AP :

(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n .

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13} .

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12} .

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10} .

(v) Given $d = 5$, $S_9 = 75$, find a and a_9 .

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n .

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d .

(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a .

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d .

(x) Given $\ell = 28$, $S = 144$, and there are total 9 terms. Find a .

Sol. (i) $a = 5, d = 3, a_n = 50$

$$\Rightarrow a + (n - 1) d = 50$$

$$\Rightarrow 5 + (n - 1) (3) = 50$$

$$\Rightarrow 5 + 3n - 3 = 50 \text{ or } 3n = 48 \text{ or } n = 16$$

$$S_{16} = \frac{16}{2} \{2a + 15d\}$$

$$= 8 \{10 + 15 \times 3\} = 440$$

(ii) $a = 7, a_{13} = 35$

$$\therefore a_{13} = a + (13 - 1) d$$

$$35 = 7 + 12 d$$

$$35 - 7 = 12d$$

$$28 = 12d$$

$$d = \frac{7}{3}$$

$$S_{13} = \frac{n}{2} [a + a_{13}]$$

$$= \frac{13}{2} [7 + 35]$$

$$= \frac{13 \times 42}{2} = 13 \times 21$$

$$= 273$$

(iii) $a_{12} = 37, d = 3$

$$a_{12} = a + (12 - 1) d$$

$$37 = a + 11d$$

$$a = 4$$

$$S_n = \frac{n}{2} [a + a_n]$$

$$S_{12} = \frac{12}{2} [4 + 37]$$

$$S_{12} = 6(41)$$

$$S_{12} = 246$$

(iv) $a_3 = 15, S_{10} = 125$

$$a_3 = a + (3 - 1) d$$

$$15 = a + 2d \quad \dots(i)$$

$$S_n = \frac{n}{2} [2a + (n - 1) d]$$

$$S_{10} = \frac{10}{2} [2a + (n - 1) d]$$

$$125 = 5(2a + 9d)$$

$$25 = 2a + 9d \quad \dots(ii)$$

On multiplying equation (i) by 2, we obtain

$$30 = 2a + 4d \quad \dots(iii)$$

On subtracting equation (iii) from (ii), we obtain

$$-5 = 5d$$

$$d = -1$$

From equation (i),

$$15 = a + 2(-1)$$

$$15 = a - 2$$

$$a = 17$$

$$a_{10} = a + (10 - 1)d$$

$$a_{10} = 17 + (9)(-1)$$

$$a_{10} = 17 - 9 = 8$$

$$(v) \ d = 5, S_9 = 75$$

$$S_9 = \frac{9}{2} [2a + (9 - 1)5]$$

$$75 = \frac{9}{2} (2a + 40)$$

$$25 = 3(a + 20)$$

$$25 = 3a + 60$$

$$3a = 25 - 60$$

$$a = \frac{-35}{3}$$

$$a_9 = a + (9 - 1)(5)$$

$$= \frac{-35}{3} + 8(5)$$

$$= \frac{-35}{3} + 40$$

$$= \frac{-35 + 120}{3} = \frac{85}{3}$$

$$(vi) \ a = 2, d = 8, S_n = 90$$

$$\Rightarrow \frac{n}{2} \{2a + (n - 1)d\} = 90$$

$$\Rightarrow \frac{n}{2} \{4 + (n - 1) \times 8\} = 90$$

$$\Rightarrow \frac{n}{2} \times \{8n - 4\} = 90$$

$$\Rightarrow 4n^2 - 2n - 90 = 0$$

$$\Rightarrow 2n^2 - n - 45 = 0$$

$$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$$

$$\Rightarrow 2n(n - 5) + 9(n - 5) = 0$$

$$\Rightarrow (n - 5)(2n + 9) = 0$$

$$\Rightarrow n - 5 = 0 \quad (\because 2n + 9 \neq 0)$$

$$\Rightarrow n = 5$$

$$a_n = a_5 = a + 4d = 2 + 4 \times 8 = 34$$

$$\Rightarrow a_n = 34$$

$$(vii) \quad a = 8, a_n = 62, S_n = 210$$

$$210 = \frac{n}{2} [8 + 62]$$

$$210 = \frac{n}{2} (70)$$

$$n = 6$$

$$a_n = a + (n - 1)d$$

$$62 = 8 + (6 - 1)d$$

$$62 - 8 = 5d$$

$$54 = 5d$$

$$d = \frac{54}{5}$$

$$(viii) \quad a_n = 4, d = 2, S_n = -14$$

$$\text{Now, } a_n = 4 \Rightarrow a + (n - 1)d = 4$$

$$\Rightarrow a + (n - 1)(2) = 4$$

$$\Rightarrow a = 6 - 2n \quad \dots(1)$$

$$S_n = -14$$

$$\Rightarrow \frac{n}{2} \{2a + (n - 1)d\} = -14$$

$$\Rightarrow \frac{n}{2} \{2(6 - 2n) + (n - 1)(2)\} = -14 \quad \{\text{By(1)}\}$$

$$\Rightarrow \frac{n}{2} \{12 - 4n + 2n - 2\} = -14$$

$$\Rightarrow \frac{n}{2} \{10 - 2n\} = -14$$

$$\Rightarrow n(n - 5) = 14$$

$$\Rightarrow n^2 - 5n - 14 = 0$$

$$\Rightarrow n^2 - 7n + 2n - 14 = 0$$

$$\Rightarrow n(n - 7) + 2(n - 7) = 0$$

$$\Rightarrow (n - 7)(n + 2) = 0$$

$$\Rightarrow n = 7$$

From (1), $a = 6 - 2 \times 7 = -8$

$$a = -8$$

$$(ix) a = 3, n = 8, S = 192$$

$$192 = \frac{8}{2} [2 \times 3 + (8 - 1)d]$$

$$192 = 4 [6 + 7d]$$

$$48 = 6 + 7d$$

$$42 = 7d$$

$$d = 6$$

$$(x) \ell = 28, \text{ i.e., } t_n = 28$$

$$\Rightarrow t_9 = 28 \Rightarrow a + 8d = 28$$

$$S = 144, \text{ i.e., } S_9 = 144$$

$$\Rightarrow \frac{9}{2} \{t_1 + t_9\} = 144 \Rightarrow \frac{9}{2} (a + 28) = 144$$

$$\Rightarrow a + 28 = 32 \Rightarrow a = 4$$

Q4. How many terms of the AP : 9, 17, 25,... must be taken to give a sum of 636?

Sol. $a = 9, d = 8$

$$\text{Let } S_n = 636$$

$$\Rightarrow \frac{n}{2} [2a + (n - 1)d] = 636$$

$$\Rightarrow \frac{n}{2} \{2 \times 9 + (n - 1)(8)\} = 636$$

$$\Rightarrow \frac{n}{2} \{18 + 8n - 8\} = 636$$

$$\Rightarrow \frac{n}{2} \{8n + 10\} = 636 \Rightarrow n(4n + 5) = 636$$

$$\Rightarrow 4n^2 + 5n - 636 = 0$$

$$\Rightarrow n = \frac{-5 \pm \sqrt{25 + 10176}}{8} = \frac{-5 \pm \sqrt{10201}}{8}$$

$$= \frac{-5 \pm 101}{8} = -\frac{106}{8} \text{ or } \frac{96}{8} = -\frac{53}{4} \text{ or } 12$$

$$\text{We reject } n = -\frac{53}{4} \Rightarrow n = 12.$$

Hence, 12 terms makes the sum.

Q5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol. $a = 5$, last term $t_n = 45$ and $S_n = 400$

$$S_n = 400 \Rightarrow \frac{n}{2} \{t_1 + t_n\} = 400$$

$$\Rightarrow \frac{n}{2} \{5 + 45\} = 400 \Rightarrow \frac{n}{2} \times 50 = 400$$

$$\Rightarrow n = 16$$

$$\text{Now, } t_n = 45 \Rightarrow t_{16} = 45$$

$$\Rightarrow a + 15d = 45 \Rightarrow 5 + 15d = 45$$

$$\Rightarrow 15d = 40 \Rightarrow d = 8/3$$

Q6. The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol. Given that,

$$a = 17$$

$$\ell = 350$$

$$d = 9$$

Let there be n terms in the A.P.

$$\ell = a + (n - 1)d$$

$$350 = 17 + (n - 1)9$$

$$333 = (n - 1)9$$

$$(n - 1) = 37$$

$$n = 38$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$\Rightarrow S_n = \frac{38}{2}(17 + 350) = 19(367) = 6973$$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Q7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Sol. $d = 7$

$$a_{22} = 149$$

$$S_{22} = ?$$

$$a_{22} = a + (22 - 1)d$$

$$149 = a + 21 \times 7$$

$$149 = a + 147$$

$$a = 2$$

$$S_n = \frac{n}{2}(a + a_n) = \frac{22}{2}(2 + 149) = 11(151) = 1661$$

Q8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol. $t_2 = 14, t_3 = 18$

$$d = t_3 - t_2 = 18 - 14 = 4, \text{ i.e., } d = 4$$

$$\text{Now } t_2 = 14 \Rightarrow a + d = 14$$

$$\Rightarrow a + 4 = 14 \Rightarrow a = 10$$

$$S_{51} = \frac{51}{2} \{2a + 50d\} = \frac{51}{2} \{2 \times 10 + 50 \times 4\}$$

$$= \frac{51}{2} \times 220 = 51 \times 110 = 5610$$

Q9. If the sum of 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of n terms.

Sol. $S_7 = 49$

$$\Rightarrow \frac{7}{2} \{2a + 6d\} = 49 \Rightarrow a + 3d = 7 \dots(1)$$

$$S_{17} = 289$$

$$\Rightarrow \frac{17}{2} \{2a + 16d\} = 289 \Rightarrow a + 8d = 17 \dots(2)$$

Subtracting (1) from (2), we get

$$5d = 17 - 7 = 10$$

$$\Rightarrow d = 2$$

From (1),

$$a + 3 \times 2 = 7$$

$$\Rightarrow a = 1$$

$$S_n = \frac{n}{2} \{2a + (n-1)d\}$$

$$= \frac{n}{2} \{2 \times 1 + (n-1) \times 2\}$$

$$= \frac{n}{2} \{2n\} = n^2$$

$$\text{Hence, } S_n = n^2$$

Q10. Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as below :

$$(i) a_n = 3 + 4n$$

$$(ii) a_n = 9 - 5n$$

Also find the sum of the first 15 terms in each case.

Sol. (i) $a_n = 3 + 4n$

Putting $n = 1, 2, 3, 4, \dots$ in (1), we get

$$a_1 = 3 + 4 = 7, a_2 = 3 + 8 = 11,$$

$$a_3 = 3 + 12 = 15, a_4 = 3 + 16 = 19, \dots$$

Thus, the sequence (list of numbers) is

$$7, 11, 15, 19, \dots$$

$$\text{Here, } a_2 - a_1 = 11 - 7 = 4$$

$$a_3 - a_2 = 15 - 11 = 4,$$

$$a_4 - a_3 = 19 - 15 = 4$$

Therefore, the sequence forms an AP in which

$$a = 7 \text{ and } d = 4.$$

$$S_{15} = \frac{15}{2} \{2a + 14d\} = \frac{15}{2} \{2 \times 7 + 14 \times 4\}$$

$$= \frac{15}{2} \times 70 = 15 \times 35 = 525$$

(ii) $a_n = 9 - 5n$

$$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$$

$$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$$

$$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$$

$$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$$

It can be observed that

$$a_2 - a_1 = -1 - 4 = -5$$

$$a_3 - a_2 = -6 - (-1) = -5$$

$$a_4 - a_3 = -11 - (-6) = -5$$

Therefore, this is an A.P. with common difference as -5 and first term as 4 .

$$S_{15} = \frac{15}{2} [2a + (n-1)d]$$

$$= \frac{15}{2} [8 + 14(-5)]$$

$$= \frac{15}{2} (8 - 70)$$

$$= \frac{15}{2} (-62) = 15(-31)$$

$$= -465$$

Q11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the n th terms.

Sol. $S_n = 4n - n^2$

Putting $n = 1$, we get $S_1 = 4 - 1 = 3$

i.e., $t_1 = 3$

$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$, i.e., $S_2 = 4$

$\Rightarrow t_1 + t_2 = 4 \Rightarrow 3 + t_2 = 4 \Rightarrow t_2 = 1$

$t_2 - t_1 = 1 - 3 = -2 \Rightarrow d = -2$

Then $t_3 = t_2 + d = 1 - 2 = -1$, i.e., $t_3 = -1$

$t_{10} = a + 9d = 3 + 9(-2) \quad (\because t_1 = a)$

$\Rightarrow t_{10} = -15$

$t_n = a + (n - 1)d = 3 + (n - 1) \times (-2)$

i.e., $t_n = 5 - 2n$

Q12. Find the sum of the first 40 positive integers divisible by 6.

Sol. The positive integers that are divisible by 6 are 6, 12, 18, 24 ...

It can be observed that these are making an A.P. whose first term is 6 and common difference is 6.

$a = 6$

$d = 6$

$S_{40} = ?$

$S_n = \frac{n}{2} [2a + (n - 1)d]$

$S_{40} = \frac{40}{2} [2(6) + (40 - 1)6]$

$= 20[12 + (39)(6)]$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

Q13. Find the sum of the first 15 multiples of 8.

Sol. The multiples of 8 are 8, 16, 24, 32...

These are in an A.P., having first term as 8 and common difference as 8.

Therefore, $a = 8$

$d = 8$

$S_{15} = ?$

$$\begin{aligned}
 S_n &= \frac{n}{2} [2a + (n-1)d] \\
 &= \frac{15}{2} [2(8) + (15-1)8] \\
 &= \frac{15}{2} [16 + 14(8)] \\
 &= \frac{15}{2} (16 + 112) \\
 &= \frac{15(128)}{2} = 15 \times 64 \\
 &= 960
 \end{aligned}$$

Q14. Find the sum of the odd numbers between 0 and 50.

Sol. 1, 3, 5, 7 ..., 49

$$a = 1, d = 2$$

$$\ell = t_n = 49$$

$$\Rightarrow a + (n-1)d = 49$$

$$\Rightarrow 1 + (n-1)(2) = 49$$

$$\Rightarrow 1 + 2n - 2 = 49$$

$$\Rightarrow 2n = 50 \text{ or } n = 25$$

$$\text{The sum} = \frac{25}{2} \{a + \ell\} = \frac{25}{2} \{1 + 49\}$$

$$= \frac{25}{2} \times 50 = 625$$

Q15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows : Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Sol. It can be observed that these penalties are in an A.P. having first term as 200 and common difference as 50.

$$a = 200$$

$$d = 50$$

Penalty that has to be paid if he has delayed the work by 30 days = S_{30}

$$S_{30} = \frac{30}{2} [2(200) + (30-1)50]$$

$$= 15 [400 + 1450]$$

$$= 15 (1850)$$

$$= 27750$$

Q16. A sum of Rs. 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, find the value of each of the prizes.

Sol. Let the 1st prize be of Rs. a .

Then the next prize will be of Rs. $(a - 20)$

Then the next prize will be of Rs. $\{(a - 20) - 20\}$,

i.e., Rs. $(a - 40)$

Thus, the seven prizes are of Rs. a , Rs. $(a - 20)$, Rs. $(a - 40)$, ... (an AP)

Then $a + (a - 20) + (a - 40) + \dots$ to 7 terms $= 700$

$$\Rightarrow \frac{7}{2} \{2a + 6 \times (-20)\} = 700 \quad (\because d = -20)$$

$$\Rightarrow \frac{7}{2} \times (2a - 120) = 700 \Rightarrow a - 60 = 100$$

$$\Rightarrow a = 160$$

Thus, the 7 prizes are of Rs. 160, Rs. 140, Rs. 120, Rs. 100, Rs. 80, Rs. 60, Rs. 40.

Q17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Sol. It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 512

First term, $a = 1$

Common difference, $d = 2 - 1 = 1$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_{12} = \frac{12}{2} [2(1) + (12 - 1)(1)]$$

$$= 6 (2 + 11)$$

$$= 6 (13)$$

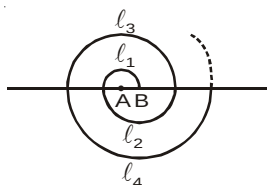
$$= 78$$

Therefore, number of trees planted by 1 section of the classes $= 78$

Number of trees planted by 3 sections of the classes $= 3 \times 78 = 234$

Therefore, 234 trees will be planted by the students.

- Q18.** A spiral is made up of successive semi-circles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,... as shown in fig. What is the total length of such a spiral made up of thirteen consecutive semi-circles? (Take $\pi = 22/7$)



Sol. From the figure,

$$l_1 = \pi \times \frac{1}{2}, l_2 = \pi \times 1, l_3 = \pi \times \frac{3}{2}, l_4 = \pi \times 2, \text{ and so. i.e., } l_1 = \frac{1}{2}\pi, l_2 = \pi, l_3 = \frac{3}{2}\pi, l_4 = 2\pi, \dots$$

Thus, $l_1, l_2, l_3, l_4, \dots$ form an AP.

$$\therefore l_2 - l_1 = l_3 - l_2 = l_4 - l_3 = \dots = \frac{1}{2}\pi$$

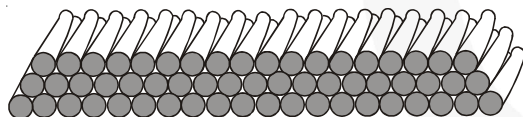
$$\text{Thus, } a = \frac{\pi}{2}, d = \frac{\pi}{2}$$

$$\text{Length of the spiral} = l_1 + l_2 + \dots + l_{13}$$

$$= \frac{13}{2} \{2a + 12d\} = \frac{13}{2} \left\{ 2 \times \frac{\pi}{2} + 12 \times \frac{\pi}{2} \right\}$$

$$= \frac{91\pi}{2} \text{ cm} = \frac{91}{2} \times \frac{22}{7} \text{ cm} = 143 \text{ cm}$$

- Q19.** 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?



Sol. It can be observed that the numbers of logs in rows are in an A.P.

20, 19, 18...

For this A.P.,

$$a = 20$$

$$d = a_2 - a_1 = 19 - 20 = -1$$

Let a total of 200 logs be placed in n rows.

$$S_n = 200$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$200 = \frac{n}{2} [2(20) + (n-1)(-1)]$$

$$400 = n(40 - n + 1)$$

$$400 = n(41 - n)$$

$$400 = 41n - n^2$$

$$n^2 - 41n + 400 = 0$$

$$n^2 - 16n - 25n + 400 = 0$$

$$n(n-16) - 25(n-16) = 0$$

$$(n-16)(n-25) = 0$$

$$\text{Either } (n-16) = 0 \text{ or } n-25 = 0$$

$$n = 16 \text{ or } n = 25$$

$$a_n = a + (n-1)d$$

$$a_{16} = 20 + (16-1)(-1)$$

$$a_{16} = 20 - 15$$

$$a_{16} = 5$$

Similarly,

$$a_{25} = 20 + (25-1)(-1)$$

$$a_{25} = 20 - 24$$

$$= -4$$

Clearly, the number of logs in 16th row is 5. However, the number of logs in 25th row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

- Q20.** In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see fig.). A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



Sol. Distance run to pick up the 1st potato

$$= 2 \times 5 = 10 \text{ m}$$

Distance run to pick up the 2nd potato

$$= 2 \times (5 + 3) \text{ m} = 16 \text{ m}$$

Distance run to pick up the 3rd potato

$$= 2 \times \{5 + 3 + 3\} \text{ m} = 22 \text{ m}$$

Thus, the sequence become 10, 16, 22, ... to 10 terms. It forms an A.P.

Here, $a = 10$, $d = 6$ and $n = 10$

$$\text{Sum} = S_{10} = \frac{10}{2} \{2a + 9d\} = 5 \times \{2 \times 10 + 9 \times 6\}$$

$$= 5 \times 74 \text{ m} = 370 \text{ m}$$

Hence, the total distance run by a competitor

$$= 370 \text{ m.}$$

