Ex - 1.4

Q1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$ (ix) $\frac{35}{50}$

(x) $\frac{77}{210}$

Sol. (i) $\frac{13}{3125} = \frac{13}{5^5}$

Hence, $q = 5^5$, which is of the form $2^n 5^m$ (n = 0, m = 5). So, the rational number $\frac{13}{3125}$ has a terminating decimal expansion.

(ii) $\frac{17}{8} = \frac{17}{2^3}$

Hence, $q = 2^3$, which is of the form $2^n 5^m$ (n = 3, m = 0). So, the rational number $\frac{17}{8}$ has a terminating decimal expansion.

(iii) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

Hence, $q = 5 \times 7 \times 13$, which is not of the form $2^n 5^m$. So, the rational number $\frac{64}{455}$ has a non-terminating repeating decimal expansion.

(iv) $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{3 \times 5}{2^6 \times 5^2}$

Hence, $q = 2^6 \times 5$, which is of the form $2^n \times 5^m$ (n = 6, m = 1). So, the rational number $\frac{15}{1600}$ has a terminating decimal expansion.

(v) $\frac{29}{343} = \frac{29}{7^3}$

Hence, $q = 7^3$, which is not of the form $2^m \times 5^n$. So, rational number $\frac{29}{7^3}$ has a non-terminating repeating decimal expansion.

(vi) $\frac{23}{2^3 \times 5^2}$

Hence, $q = 2^3 \times 5^2$,

which is of the form $2^n \times 5^m$ (n = 3, m = 2). So, the rational number has a terminating decimal

expansion.

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

Hence, q = $2^2 \times 5^7 \times 7^5$, which is not of the form $2^m \times 5^n$. So, rational number $\frac{129}{2^2 \times 5^7 \times 7^5}$ has a non-terminating repeating decimal expansion.

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$

Hence, q = 5, which is of the form $2^m \times 5^n$ (m = 0, n = 1). So, the rational no $\frac{2}{5}$ has a terminating decimal expansion.

(ix) $\frac{35}{50} = \frac{5 \times 7}{2 \times 5^2} = \frac{7}{2 \times 5}$

Hence, q = 2×5^1, which is of the form $2^m \times 5^n$ (m = 1, n = 1). So, the rational number $\frac{35}{50}$

has a terminating decimal expansion.

(x) $\frac{77}{210} = \frac{7 \times 11}{2 \times 3 \times 5 \times 7} = \frac{11}{2 \times 3 \times 5}$

Hence, q = $2 \times 3 \times 5$, which is not of the form $2^m \times 5^n$. So, the rational number $\frac{77}{210}$ has a

non-terminating repeating decimal expansion.

Q2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Sol. (i) $\frac{13}{3125} = \frac{13}{5^5} = \frac{13 \times 2^5}{5^5 \times 2^5} = \frac{416}{10^5} = 0.00416$

(ii) $\frac{17}{8} = \frac{17}{2^3}$

$= \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 5^3}{10^3} = \frac{2125}{10^3} = 2.125$

(iv) $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{3 \times 5}{2^6 \times 5^2} = \frac{3}{2^6 \times 5}$

$= 0.009375$

(vi) $\frac{23}{2^3 \times 5^2} = 0.115$

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5} = 0.4$

(ix) $\frac{35}{50} = \frac{5 \times 7}{2 \times 5^2} = \frac{7}{2 \times 5} = 0.7$

Q3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational, or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120 1200 12000 120000....

(iii) $43.\overline{123456789}$

Sol. (i) 43.123456789

Since, the decimal expansion terminates, so the given real number is rational and therefore of the form $\frac{p}{q}$. 43.123456789

$= \frac{43123456789}{1000000000}$

$= \frac{43123456789}{10^9}$

$= \frac{43123456789}{(2 \times 5)^9}$

$= \frac{43123456789}{2^9\ 5^9}$

Hence, $q = 2^9\ 5^9$

The prime factorization of q is of the form $2^n\ 5^m$, where n = 9, m = 9.

(ii) 0.120 1200 12000 120000....

Since, the decimal expansion is neither terminating nor non-terminating repeating, therefore, the given real number is not rational.

(iii) $43.\overline{123456789}$

Since, the decimal expansion is non-terminating and repeating, therefore, the given real number

is rational.

As the number is non-terminating so q is not of the form $2^m \times 5^n$.