Ex - 8.2

Q1. ABCD is a quadrilateral in which P, Q, R and S are mid points of the sides AB, BC, CD and DA (fig.) AC is a diagonal. Show that

(i) SR||AC and SR = 1/2 C

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Sol. **Given :** ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA. AC is a diagonal.

To prove : (i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Proof : (i) In ΔDAC,

∵ S is the mid-point of DA and R is the mid-point of DC

∴ SR || AC and SR = $\frac{1}{2}$ AC [By Mid-point theorem]

(ii) In ΔBAC,

∵ P is the mid-point of AB and Q is the mid-point of BC

∴ PQ || AC and PQ = $\frac{1}{2}$ AC

[By Mid-point theorem]

But from (i) SR = $\frac{1}{2}$ AC & (ii) PQ = $\frac{1}{2}$ AC

⇒ PQ = SR

(iii) PQ || AC [From (ii)]

SR || AC [From (i)]

∴ PQ || SR

[Two lines parallel to the same line are parallel to each other]

Also, PQ = SR [From (ii)]

∴ PQRS is a parallelogram.

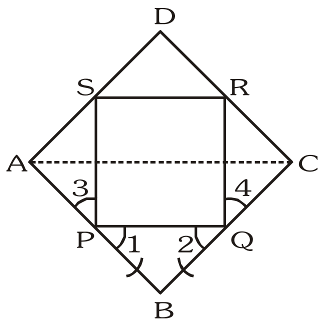
[A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

Q2. ABCD is a rhombus and P, Q, R and S are the mid points of sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Sol. **Given :** P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.

To prove : PQRS is a rectangle.

Construction : Join A and C.



Proof : In ΔABC, P is the mid-point of AB and Q is the mid-point of BC.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ .....(i)

In ΔADC, R is the mid-point of CD and S is the mid-point of AD.

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2}AC$ ....(ii)

From eq. (i) and (ii), $PQ \parallel SR$ and $PQ = SR$

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rhombus [Given]

$\therefore$ $AB = BC$

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}BC \Rightarrow PB = BQ$

$\therefore$ $\angle 1 = \angle 2$ [Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

$AP = CQ$ [P and Q are the mid-points of AB and BC and AB = BC]

Similarly, AS = CR and PS = QR

[Opposite sides of a parallelogram]

$\therefore$ $\Delta APS \cong \Delta CQR$ [By SSS congruency]

$\Rightarrow$ $\angle 3 = \angle 4$ [By C.P.C.T.]

Now, we have $\angle 1 + \angle SPQ + \angle 3 = 180°$

and $\angle 2 + \angle PQR + \angle 4 = 180°$

$\therefore$ $\angle 1 + \angle SPQ + \angle 3 = \angle 2 + \angle PQR + \angle 4$

Since $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Proved above]

$\therefore$ $\angle SPQ = \angle PQR$ .....(iii)

Now PQRS is a parallelogram [Proved above]

$\therefore$ $\angle SPQ + \angle PQR = 180°$ .....(iv)

[Interior angles]

Using eq. (iii) and (iv),

$\angle SPQ + \angle SPQ = 180°$

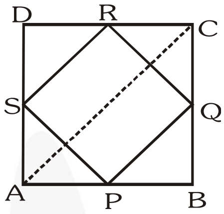
$\Rightarrow$ $2\angle SPQ = 180° \Rightarrow \angle SPQ = 90°$

Hence, PQRS is a rectangle.

Q3. ABCD is a rectangle and P,Q,R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Sol. **Given :** A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joinned.

To prove : PQRS is a rhombus.



Construction : Join AC.

Proof : In Δ ABC, P and Q are the mid-points of sides AB, BC respectively.

$\therefore \quad PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ ....(i)

In Δ ADC, R and S are the mid-points of sides CD, AD respectively.

$\therefore \quad SR \parallel AC$ and $SR = \frac{1}{2} AC$ ...(ii)

From eq.(i) and (ii), $PQ \parallel SR$ and $PQ=SR$ ...(iii)

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rectangle. [Given]

$\therefore \quad AD = BC$

$\Rightarrow \quad \frac{1}{2} AD = \frac{1}{2} BC \Rightarrow AS = BQ$ ...(iv)

In triangles APS and BPQ,

$AP = BP$ [P is the mid-point of AB]

$\angle PAS = \angle PBQ$ [Each 90°]

and $AS = BQ$ [From eq. (iv)]

$\therefore \quad \Delta APS \cong \Delta BPQ$ [By SAS congruency]

$\Rightarrow \quad PS = PQ$ [By C.P.C.T.] .....(v)

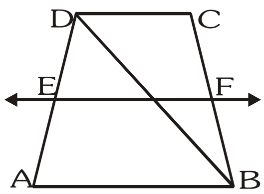
From eq.(iii) and (v), we get that PQRS is a parallelogram.

$\Rightarrow \quad PS = PQ$

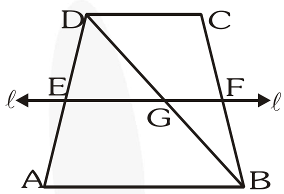
$\Rightarrow$ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

Q4. ABCD is a trapezium in which AB||DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (fig.). Show that F is the mid-point of BC.



Sol. Line ℓ || AB and passes through E.



Line ℓ meets BC in F and BD in G.

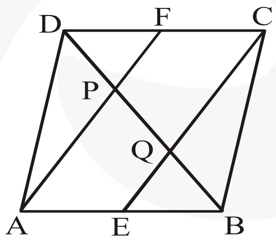
In ΔABD, E is mid-point of AD and EG || AB.

$\Rightarrow$ G is mid-point of BD.

Also, ℓ || AB and AB || CD $\Rightarrow$ ℓ || CD

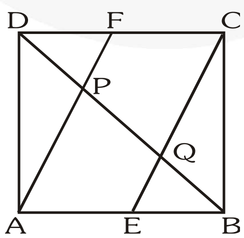
$\Rightarrow$ F is mid-point of BC. [$\because$ G is mid-point of BD]

Q5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (fig.). Show that the line segments AF and EC trisect the diagonal BD.



Sol. Since E and F are the mid-points of AB and CD respectively.

Given : ABCD is a parallelogram. E and F are midpoints of AB and AC respectively.



To prove : DP = PQ = QB

Proof : -

$\therefore \quad AE = \frac{1}{2}AB$ and $CF = \frac{1}{2}CD$ ....(i)

But ABCD is a parallelogram.

$\therefore \quad AB = CD$ and $AB \parallel DC$

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}CD$ and $AB \parallel DC$

$\Rightarrow \quad AE = FC$ and $AE \parallel FC$ [From eq. (i)]

$\therefore \quad$ AECF is a parallelogram.

$\Rightarrow \quad FA \parallel CE \quad \Rightarrow FP \parallel CQ$

[FP is a part of FA and CQ is a part of CE] ..... (ii)

Since the line segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In ΔDCQ, F is the mid-point of CD and

$\Rightarrow \quad FP \parallel CQ$

$\therefore \quad$ P is the is mid-point of DQ.

$\Rightarrow \quad DP = PQ$ ....(iii)

Similarly, In ΔABP, E is the mid-point of AB and

$\Rightarrow \quad EQ \parallel AP$

$\therefore \quad$ Q is the mid-point of BP.

$\Rightarrow \quad BQ = PQ$ ....(iv)

From eq.(iii) and (iv),

$DP = PQ = BQ$ ....(v)

Now, $BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ$

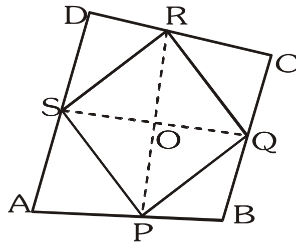
$\Rightarrow \quad BQ = \frac{1}{3}BD$ .....(vi)

From eq (v) and (vi), $DP = PQ = BQ = \frac{1}{3}BD$

$\Rightarrow \quad$ Points P and Q trisects BD. So AF and CE trisects BD.

Q6. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Sol. P,Q,R and S are the mid-points of the sides AB, BC, CD and AD of the quadrilateral ABCD.



We have to prove that, PR and QS bisects each other. Now, join PQ, QR, RS and PS.

Here, we can prove that PQRS is a parallelogram (as in solution).

Now, PR and QS are the diagonals of the parallelogram PQRS.

Hence, PR and QS bisect each other at O.

Q7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

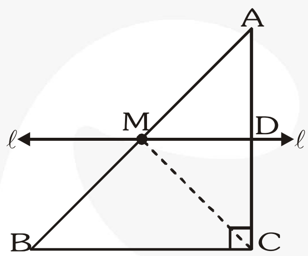
(i) D is the mid-point of AC

(ii) MD $\perp$ AC

(iii) CM = MA = 1/2 AB

Sol. (i) Through M, we draw line ℓ || BC. ℓ intersects AC at D.

$\Rightarrow$ D is mid-point of AC.



(ii) $\angle ADM = \angle ACB = 90°$

[Corresponding angles]

$\Rightarrow$ $\angle ADM = 90°$ $\Rightarrow$ MD $\perp$ AC.

(iii) In ΔCMD and ΔAMD;

CD = AD, MD = MD

and $\angle CDM = \angle ADM$ [Each = 90°]

Therefore, $\Delta CMD \cong \Delta AMD$

$\Rightarrow$ CM = AM; Also AM = 1/2 AB.