Ex - 4.2

Q1. Find the roots of the following quadratic equations by factorisation :

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Sol. (i) $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x + 2)(x - 5) = 0$

$\Rightarrow x + 2 = 0$ or $x - 5 = 0$

$\Rightarrow x = -2$ or $x = 5$

Hence, the two roots are -2 and 5.

(ii) $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 3) = 0$

$\Rightarrow x + 2 = 0$ or $2x - 3 = 0$

$\Rightarrow x = -2$ or $x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$\Rightarrow x = -\sqrt{2}$ or $-\frac{5}{\sqrt{2}}$

Hence, the two roots are $-\sqrt{2}$ and $-\frac{5}{\sqrt{2}}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

or $16x^2 - 8x + 1 = 0$

or $(4x - 1)^2 = 0$

$\Rightarrow$ Both roots are given by $4x - 1 = 0$,

i.e., $x = \frac{1}{4}$. Hence, the roots are $\frac{1}{4}, \frac{1}{4}$.

(v) $100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow (10x - 1)^2 = 0$

$\Rightarrow (10x - 1) = 0$ or $(10x - 1) = 0$

$\Rightarrow x = \frac{1}{10}$ or $x = \frac{1}{10}$

Q2. Represent the following situations mathematically.

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

Sol. (i) Let the number of marbles John had be x.

Then the number of marbles Jivanti had

= 45 – x (Why?).

The number of marbles left with John, when he lost 5 marbles = x – 5.

The number of marbles left with Jivanti, when she lost 5 marbles = 45 – x – 5 = 40 – x.

Therefore, the is product = (x – 5) (40 – x)

$= 40x - x^2 - 200 + 5x = -x^2 + 45x - 200$

So, $-x^2 + 45x - 200 = 124$

(Given that product = 124)

i.e., $-x^2 + 45x - 324 = 0$

i.e., $x^2 - 45x + 324 = 0$

Therefore, the number of marbles John had, satisfies the quadratic equation.

$x^2 - 45x + 324 = 0$

which is the required representation of the problem mathematically.

(ii) Let the number of toys produced be x.

$\therefore$ Cost of production of each toy = Rs (55 – x)

It is given that, total cost of production of the toys = Rs 750

$\therefore \quad x(55 - x) = 750$

Therefore, $x^2 - 55x + 750 = 0$

which is the required representation of the problem mathematically.

Q3. Find two numbers whose sum is 27 and product is 182.

Sol. Let one number be x, then second number = 27 – x

$x \times (27 - x) = 182$

$\Rightarrow 27x - x^2 = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 14x - 13x + 182 = 0$

$\Rightarrow$ x (x – 14) – 13 (x – 14) = 0

$\Rightarrow$ (x – 13) (x – 14) = 0

$\Rightarrow$ x = 13 or 14

$\Rightarrow$ 27x = 14 or 13

Hence, the two marbles are 13 and 14.

Q4. Find two consecutive positive integers, sum of whose squares is 365.

Sol. Let the consecutive positive integers be x and x + 1.

Given that $x^2 + (x + 1)^2 = 365$

$\Rightarrow$ $x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow$ $2x^2 + 2x – 364 = 0$

$\Rightarrow$ $x^2 + x – 182 = 0$

$\Rightarrow$ $x^2 + 14x – 13x – 182 = 0$

$\Rightarrow$ x(x + 14) – 13(x + 14) = 0

$\Rightarrow$ (x + 14)(x – 13) = 0

Either x + 14 = 0 or x – 13 = 0,

i.e., x = –14 or x = 13

Since the integers are positive, x can only be 13.

$\therefore$ x + 1 = 13 + 1 = 14

Therefore, two consecutive positive integers will be 13 and 14.

Q5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

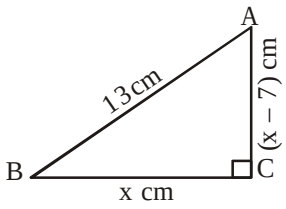
Sol. In ΔABC, base BC = x cm

and altitude AC = (x – 7) cm

$\angle ACB = 90°$

AB = 13 cm

By Pythagoras theorem, we have



$BC^2 + AC^2 = AB^2$

$\Rightarrow$ $x^2 + (x – 7)^2 = 13^2$

$\Rightarrow$ $x^2 + x^2 – 14x + 49 = 169$

$\Rightarrow$ $2x^2 – 14x – 120 = 0$

$\Rightarrow$ $x^2 – 7x – 60 = 0$

$\Rightarrow$ $x^2 - 12x + 5x - 60 = 0$

$\Rightarrow$ $x(x - 12) + 5(x - 12) = 0$

$\Rightarrow$ $(x + 5)(x - 12) = 0$

$\Rightarrow$ $x = -5$ or $x = 12$

We reject $x = -5$

$\Rightarrow$ $x = 12$

Therefore, BC = 12 cm and AC = 5 cm.

Q6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production of that day was Rs. 90, find the number of articles produced and the cost of each article.

Sol. Let the number of articles produced be x.

Therefore, cost of production of each article = Rs (2x + 3)

It is given that the total production is Rs 90.

$\therefore$ $x(2x + 3) = 90$

$\Rightarrow$ $2x^2 + 3x - 90 = 0$

$\Rightarrow$ $2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow$ $x(2x + 15) - 6(2x + 15) = 0$

$\Rightarrow$ $(2x + 15)(x - 6) = 0$

Either $2x + 15 = 0$ or $x - 6 = 0$,

i.e., $x = \frac{-15}{2}$ or $x = 6$

As the number of articles produced can only be a positive integer,

therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = $2 \times 6 + 3$ = Rs. 15