

EX-2.3

Q1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$.

Sol. (i) $x^2 - 2 \overline{) x^3 - 3x^2 + 5x - 3} \quad q(x) = (x - 3)$

$$\begin{array}{r} x^3 \qquad \qquad - 2x \\ - \qquad \qquad \qquad + \\ \hline - 3x^2 + 7x - 3 \\ - 3x^2 \qquad \qquad + 6 \\ + \qquad \qquad \qquad - \\ \hline r(x) = (7x - 9) \end{array}$$

Hence, Quotient $q(x) = x - 3$ and Remainder $r(x) = 7x - 9$

(ii) $x^2 - x + 1 \overline{) x^4 - 3x^2 + 4x + 5} \quad q(x) = x^2 + x - 3$

$$\begin{array}{r} x^4 - 3x^2 + 4x + 5 \\ - x^4 + x^2 \qquad \qquad \mp x^3 \\ \hline x^3 - 4x^2 + 4x + 5 \\ - x^3 + x^2 + x \\ \hline - 3x^2 + 3x + 5 \\ - 3x^2 + 3x - 3 \\ + \qquad \qquad \qquad + \\ \hline r(x) = 8 \end{array}$$

Hence, Quotient, $q(x) = x^2 + x - 3$
and remainder, $r(x) = 8$

(iii) $-x^2 + 2 \overline{) x^4 - 5x + 6} \quad q(x) = -x^2 - 2$

$$\begin{array}{r} x^4 - 5x + 6 \\ - x^4 \qquad \qquad \qquad \mp 2x^2 \\ \hline 2x^2 - 5x + 6 \\ 2x^2 \qquad \qquad \mp 4 \\ \hline r(x) = -5x + 10 \end{array}$$

Hence, Quotient, $q(x) = -x^2 - 2$
Remainder, $r(x) = -5x + 10$

Q2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

(i) $t^2 - 3$, $2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

Sol. (i) $q(t) = 2t^2 + 3t + 4$

$$\begin{array}{r}
 t^2 - 3 \overline{) 2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
 \underline{2t^4 - 6t^2} \\
 3t^3 + 4t^2 - 9t - 12 \\
 \underline{3t^3 - 9t} \\
 4t^2 - 12 \\
 \underline{4t^2 - 12} \\
 \hline
 \text{Remainder} = 0
 \end{array}$$

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $q(x) = 3x^2 - 4x + 2$

$$\begin{array}{r}
 x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 \underline{3x^4 + 9x^3 + 3x^2} \\
 -4x^3 - 10x^2 + 2x + 2 \\
 \underline{+ 4x^3 + 12x^2 + 4x} \\
 2x^2 + 6x + 2 \\
 \underline{2x^2 + 6x + 2} \\
 \hline
 \text{Remainder} = 0
 \end{array}$$

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $q(x) = x^2 - 1$

$$\begin{array}{r}
 x^3 - 3x + 1 \overline{) x^5 - 4x^3 + x^2 + 3x + 1} \\
 \underline{x^5 - 3x^3 + x^2} \\
 -x^3 + 3x + 1 \\
 \underline{-x^3 + 3x - 1} \\
 \hline
 \text{Remainder} = 2
 \end{array}$$

Hence, $x^2 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$

Q3. Obtain all other zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Two of the zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)$$

is a factor of the polynomial.

i.e., $x^2 - \frac{5}{3}$ is a factor.

i.e., $(3x^2 - 5)$ is a factor of the polynomial. Then we apply the division algorithm as below :

$$\begin{array}{r} \text{q (x)} = x^2 + 2x + 1 \\ 3x^2 - 5 \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\ \underline{3x^4 \quad - 5x^2} \\ 6x^3 + 3x^2 - 10x - 5 \\ \underline{6x^3 \quad - 10x} \\ 3x^2 - 5 \\ \underline{3x^2 - 5} \\ 0 \end{array}$$

The other two zeros will be obtained from the quadratic polynomial $q(x) = x^2 + 2x + 1$

Now $x^2 + 2x + 1 = (x + 1)^2$.

Its zeros are $-1, -1$.

Hence, all other zeros are $-1, -1$.

Q4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Sol. $(x^3 - 3x^2 + x + 2) = g(x) \times (x - 2) + (-2x + 4)$
 $\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = g(x) \times (x - 2)$
 $\Rightarrow x^3 - 3x^2 + 3x - 2 = g(x) \times (x - 2)$

$$g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

$$\begin{array}{r} x^2 - x + 1 \\ x-2 \overline{) x^3 - 3x^2 + 3x - 2} \\ \underline{-x^3 + 2x^2} \\ -x^2 + 3x - 2 \\ \underline{+x^2 - 2x} \\ x - 2 \\ \underline{-x + 2} \\ 0 \end{array}$$

So, $g(x) = x^2 - x + 1$

Q5. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$

(ii) $\deg q(x) = \deg r(x)$

(iii) $\deg r(x) = 0$.

Sol. (i) $p(x) = 2x^2 + 2x + 8$, $g(x) = 2x^0 = 2$;

$q(x) = x^2 + x + 4$; $r(x) = 0$

(ii) $p(x) = 2x^2 + 2x + 8$; $g(x) = x^2 + x + 9$;

$q(x) = 2$; $r(x) = -10$

(iii) $p(x) = x^3 + x + 5$; $g(x) = x^2 + 1$;

$q(x) = x$; $r(x) = 5$.