Ex - 3.5

Q1. Which of the following pairs of linear equations has unique solution, no solution, or infi nitely many solutions. In case there is a unique solution, find it by using cross multiplication method.

(i) $x - 3y - 3 = 0$
$3x - 9y - 2 = 0$

(ii) $2x + y = 5$
$3x + 2y = 8$

(iii) $3x - 5y = 20$
$6x - 10y = 40$

(iv) $x - 3y - 7 = 0$
$3x - 3y - 15 = 0$

Sol. (i) $x - 3y - 3 = 0,\ 3x - 9y - 2 = 0$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{3}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, no solution.

(ii) $2x + y = 5$ ...(i) and $3x + 2y = 8$ ...(ii)

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \left(\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2}\right)$$

Here, we have a unique solution. By cross multiplication, we have

$$\frac{x}{\begin{vmatrix} 1 & -5 \\ 2 & -8 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -5 & 2 \\ -8 & 3 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}}$$

$$\Rightarrow \frac{x}{\{(1)(-8)-(2)(-5)\}} = \frac{y}{\{(-5)(3)-(-8)(2)\}}$$

$$= \frac{1}{\{(2)(2)-(3)(1)\}}$$

$$\Rightarrow \frac{x}{(-8+10)} = \frac{y}{(-15+16)} = \frac{1}{(4-3)}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{1}{1} \quad \Rightarrow \frac{x}{2} = \frac{1}{1} \text{ and } \frac{y}{1} = \frac{1}{1}$$

$\Rightarrow x = 2$ and $y = 1$

(iii) $3x - 5y = 20$ ....(i)
$6x - 10y = 40$ ....(ii)

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{20}{40} = \frac{1}{2}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, infinite solutions

(iv) $x - 3y - 7 = 0$ ....(i)

$3x - 3y - 15 = 0$ ....(ii)

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = 1, \frac{c_1}{c_2} = \frac{7}{15}$$

$$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, unique solution.

$$\frac{x}{(-3)(-15)-(-3)(-7)} = \frac{y}{3\times(-7)-1\times(-15)}$$

$$= \frac{1}{1\times(-3)-3(-3)}$$

$$\Rightarrow \frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9}$$

$$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$$x = \frac{24}{6} = 4, \quad y = \frac{-6}{6} = -1$$

Q2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions? $2x + 3y = 7$ $(a - b)\,x + (a + b)\,y = 3a + b - 2$

(ii) For which value of k will the following pair of linear equations have no solution? $3x + y = 1$ $(2k - 1)\,x + (k - 1)\,y = 2k + 1$.

Sol. (i) $2x + 3y - 7 = 0$ ...(i)

$(a - b)\,x + (a + b)\,y - (3a + b - 2) = 0$ ...(ii)

For infinite number of solutions, we have

$$\frac{a-b}{2} = \frac{a+b}{3} = \frac{3a+b-2}{7}$$

For first and second, we have

$\dfrac{a-b}{2} = \dfrac{a+b}{3}$ or $3a - 3b = 2a + 2b$

or $a = 5b$ ...(i)

From second and third, we have

$$\frac{a+b}{3} = \frac{3a+b-2}{7}$$

or $7a + 7b = 9a + 3b - 6$ or $4b = 2a - 6$

or $2b = a - 3$ ...(ii)

From (i) and (ii), eliminating a,

$2b = 5b - 3 \Rightarrow b = 1$

Substituting $b = 1$ in (i), we get $a = 5$

(ii) $3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

For no, solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

So, $\frac{3}{2k-1} = \frac{1}{k-1}$ & $\frac{1}{k-1} \neq \frac{1}{2k+1}$

$3(k - 1) = 2k - 1$ $2k + 1 \neq k - 1$

$k = 2$ $k \neq -2$

Q3. Solve the following pair of linear equations by the substitution and cross-multiplication methods: $8x + 5y = 9$, $3x + 2y = 4$

Sol. By substitution method,

$8x + 5y = 9$ ...(i)

$3x + 2y = 4$ ...(ii)

From (ii), we get

$x = \frac{4-2y}{3}$

Substituting x from (iii) in (i), we get

$8\left(\frac{4-2y}{3}\right) + 5y = 9$

$\Rightarrow 32 - 16y + 15y = 27$

$\Rightarrow 5 = y$

Substituting y = 5 in (ii) we get

$3x + 2(v) = 4$

$\Rightarrow 3x = -6$

$\Rightarrow x = -2$

Hence, $x = -2$, $y = 5$

By cross multiplication method

$8x + 5y = 9$ ...(i)

$3x + 2y = 4$ ...(ii)

$$\frac{x}{5\times(-4)-2(-9)} = \frac{y}{3\times(-9)-8(-4)} = \frac{1}{8\times 2-3\times 5}$$

$\Rightarrow x = \frac{-20+18}{1} = -2$

$\Rightarrow y = \frac{-27+32}{1} = 5$

Hence, $x = -2$, $y = 5$

Q4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method.

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs. 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs. 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Sol. (i) Let the fixed charge be x

and charges of food for 1 day be y.

So, $x + 20y = 1000$ ....(i)

$x + 26y = 1180$ ....(ii)

Subtracting (i) from (ii), we get

$6y = 180$; $y = 30$

Substituting y in (i), we get

$x + 20 \times 30 = 1000$

$x = 1000 - 600$

$x = 400$

So, fixed charge = Rs.400 and charges of food for 1 day = Rs.30

(ii) Let the fraction be $\frac{x}{y}$

Then, $\frac{x-1}{y} = \frac{1}{3}$ ....(i)

$\frac{x}{y+8} = \frac{1}{4}$ ....(ii)

From (i) and (ii), we get

$3x - 3 = y$ or $3x - y = 3$ ......(iii)

$4x = y + 8$ or $4x - y = 8$ ......(iv)

Subtracting (iii) from (iv), we get

$4x - y - 3x + y = 5$

$x = 5$

Substituting x in (iii), we get

$3 \times 5 - y = 3$

$y = 12$

So the required fraction is $\frac{5}{12}$

(iii) Number of right answers = x. Number of wrong answers = y

Then, $3x - y = 40$ ...(i)

$4x - 2y = 50$ ...(ii)

Multiplying (i) by 2, we get

$6x - 2y = 80$ ...(iii)

Subtracting (ii) from (iii), we get

$2x = 30$

$\Rightarrow x = 15$

Substituting x = 15, in (i), we get

$45 - y = 40$

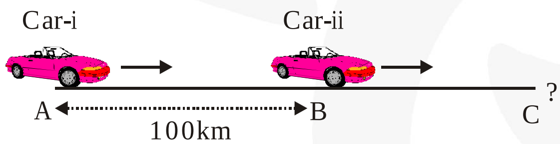
$\Rightarrow y = 5$

Total questions = x + y = 15 + 5 = 20

(iv) Speed of car i = x km/hr

Speed of car ii = y km/hr

First case :

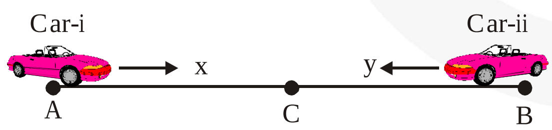


Two cars meet at C after 5 hrs.

AC – BC = AB

$\Rightarrow 5x - 5y = 100$ ...(i)

Second case :



Two cars meet at C after one hour

$x + y = 100$ ...(ii)

Multiplying (ii) by 5, we get

$5x + 5y = 500$ ...(iii)

Adding (i) and (iii), we get

$10x = 600$

$\Rightarrow x = 60$ km/hr

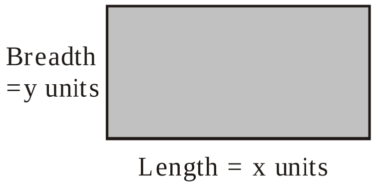
Substituting $x = 60$ km/hr in (ii), we get

$y = 40$ km/hr

Speed of car (i) = 60 km/hr

Speed of car (ii) 40 km/hr

(v) In first case, area is reduced by 9 square units.



When length = $x - 5$ units

and breadth = $y + 3$ units

$\Rightarrow xy - (x - 5) \times (y + 3) = 9$ ...(i)

In second case area increases by 67 sq. units when length = $x + 3$ and breadth = $y + 2$.

$\Rightarrow (x + 3) \times (y + 2) - xy = 67$ ...(ii)

Solving both equations, we get

$xy - xy - 3x + 5y + 15 = 9$

$3x - 5y = 6$ ...(iii)

$xy + 2x + 3y + 6 - xy = 67$

$2x + 3y = 61$ ...(iv)

Multiplying (iii) by 3 and (iv) by 5, we get

$9x - 15y = 18$

$10x + 15y = 305$

Adding, we get

$19x = 323$

$\Rightarrow x = 17$

By putting $x = 17$, we get $y = 9$